

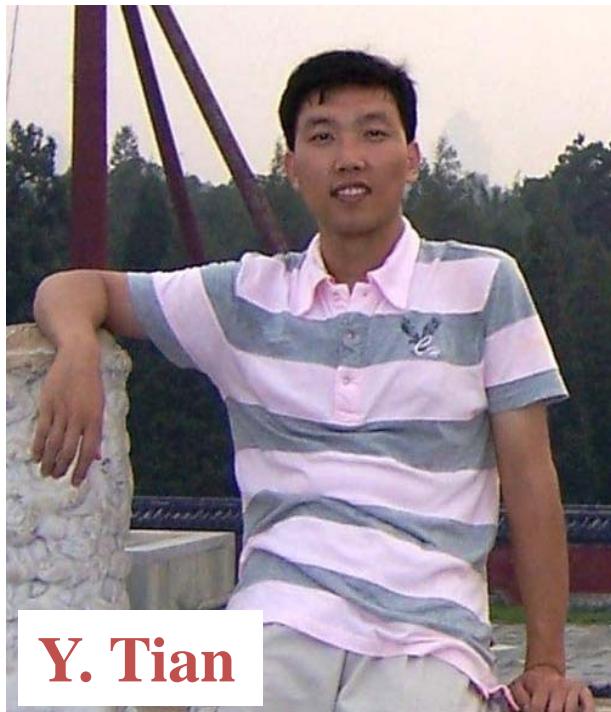
Temperature Dependence of the Intrinsic Anomalous Hall Effect in Ni

Xiaofeng Jin

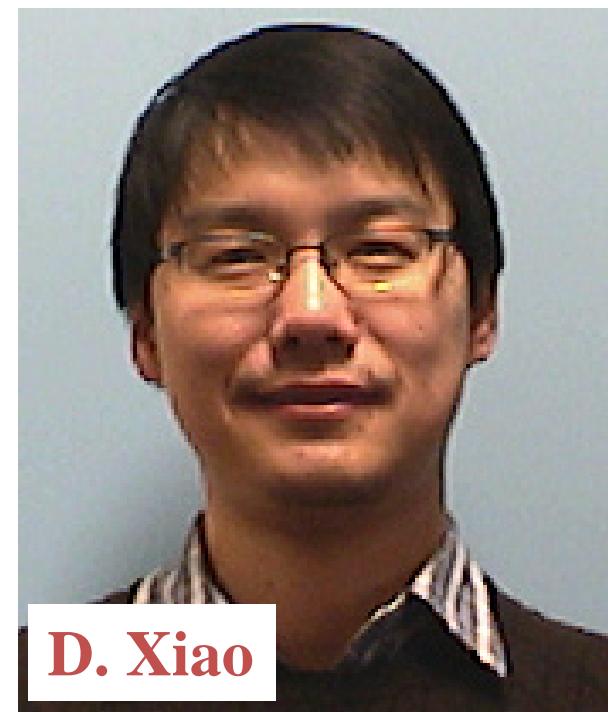
Physics Department, Fudan University, Shanghai, China



L. Ye



Y. Tian



D. Xiao

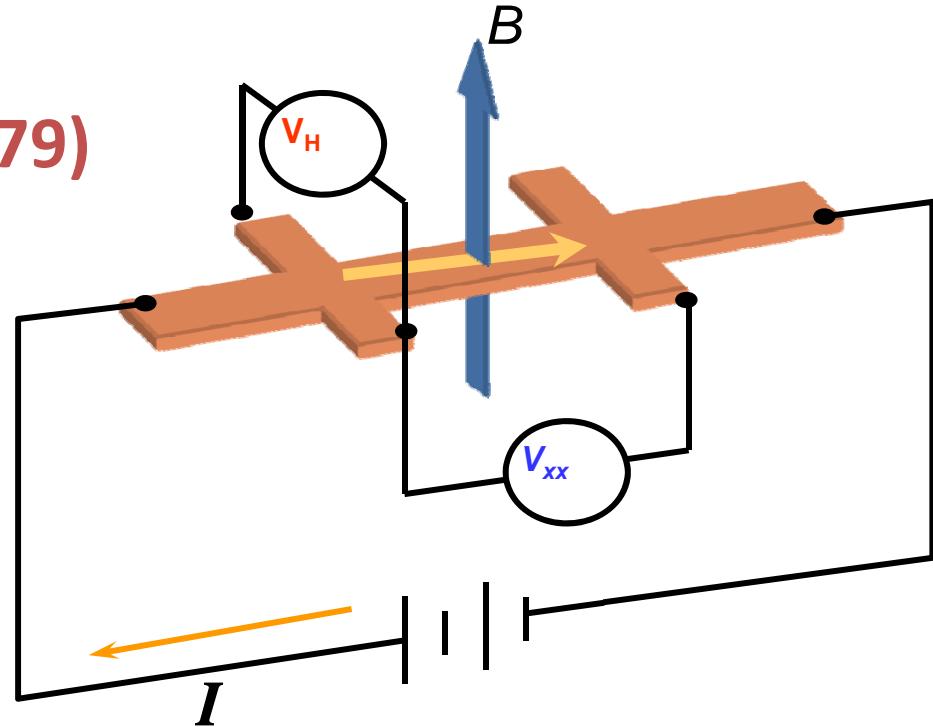
1. Introduction

- Ordinary Hall effect (1879)

$$\rho_{xy} = R_0 B$$

- Anomalous Hall effect (1880&1881)

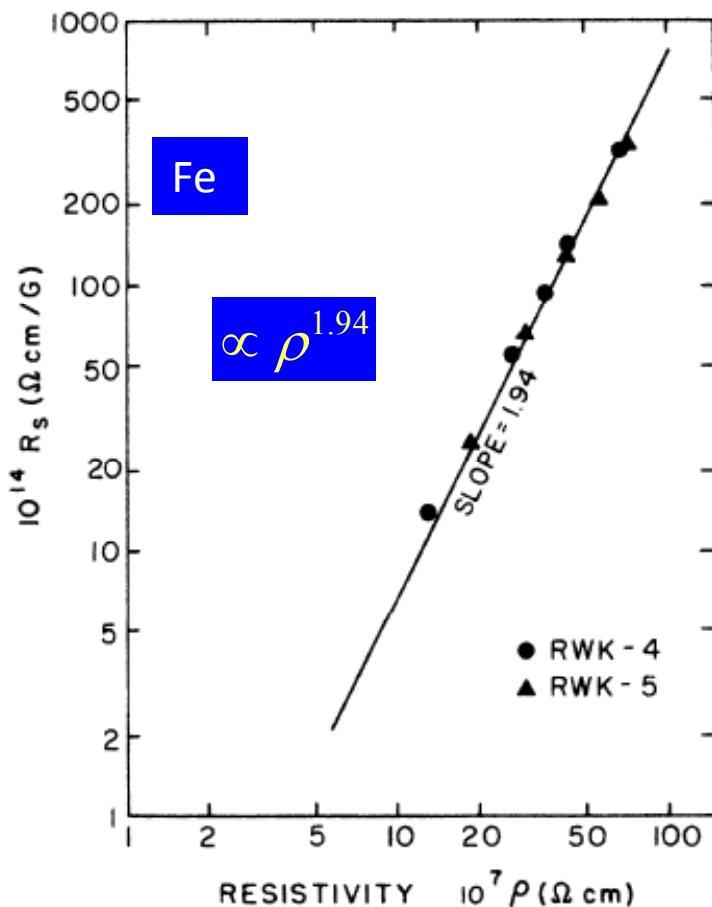
$$\rho_{xy} = R_0 B + \rho_{ah}$$



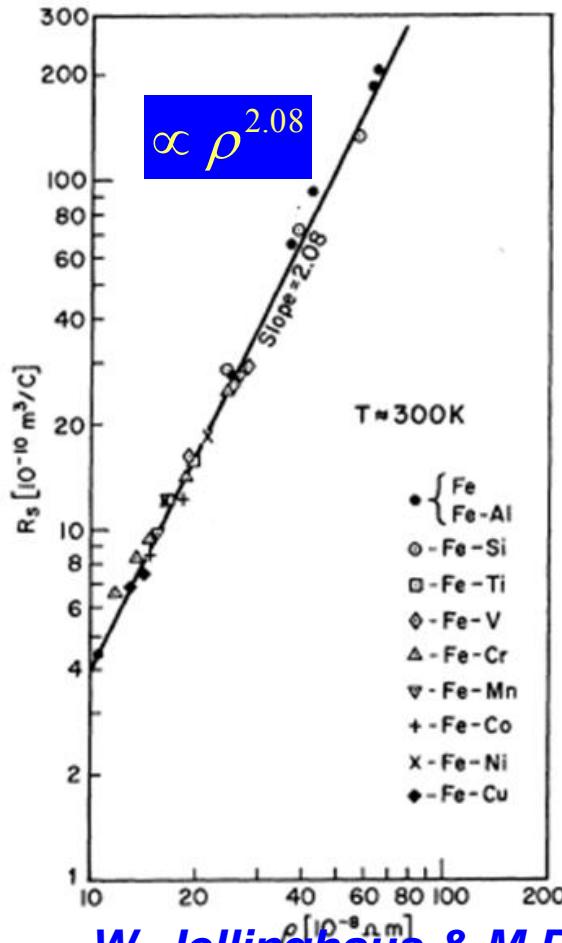
$$\rho_{ah} = f(\rho_{xx})$$

Category 1:

$$b\rho_{xx}^2$$



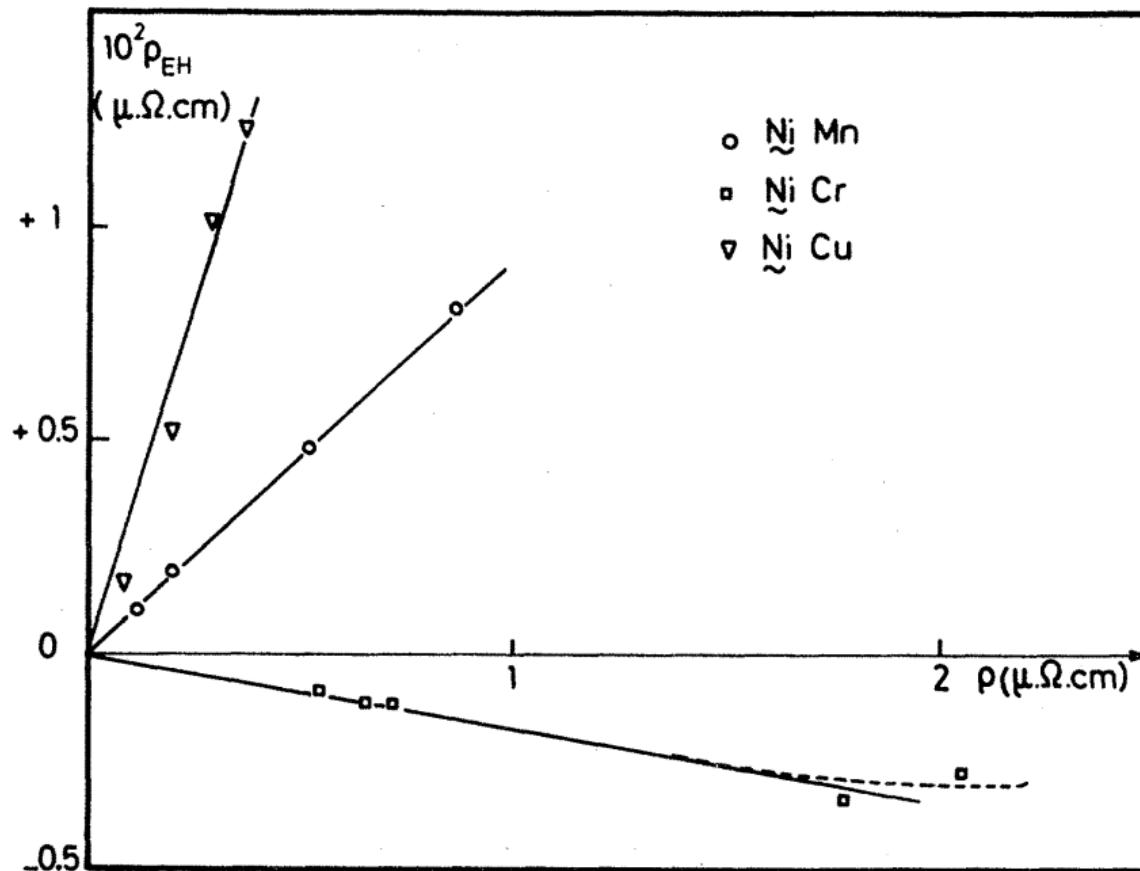
R. W. Klaffy & R. V. Coleman,
Phy. Rev B. 10, 2915 (1974).



W. Jellinghaus & M.P. DeAndres,
Ann. Physik, 462, 189 (1961)

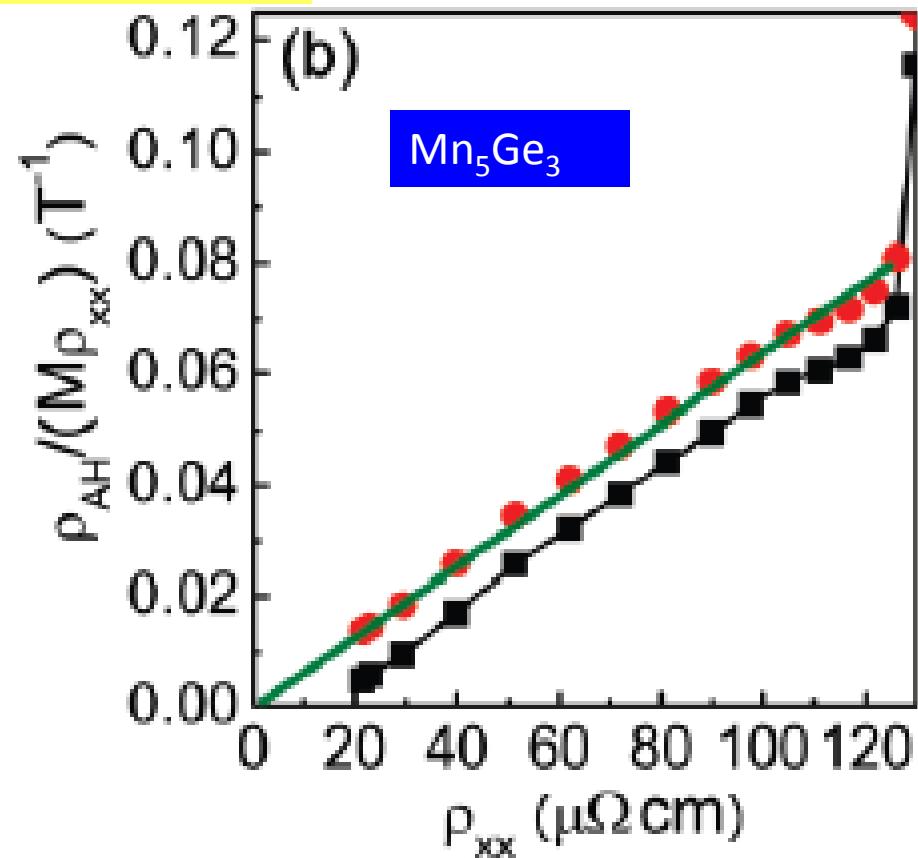
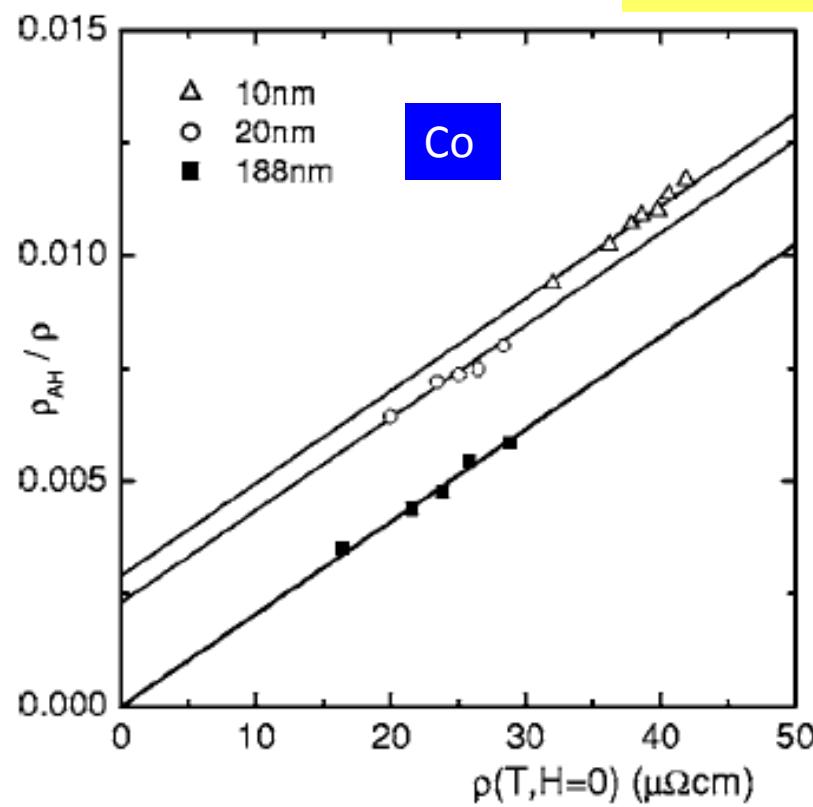
Category 2:

$$a\rho_{xx}$$



Category 3:

$$a\rho_{xx} + b\rho_{xx}^2$$

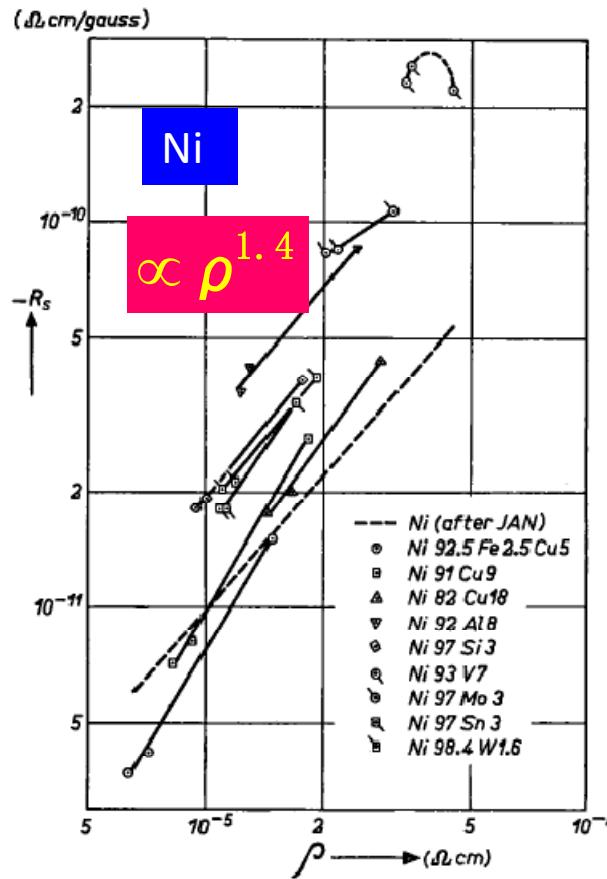


J. Kotzler and W. Gil,
Phys. Rev. B 72, 060412 (2005)

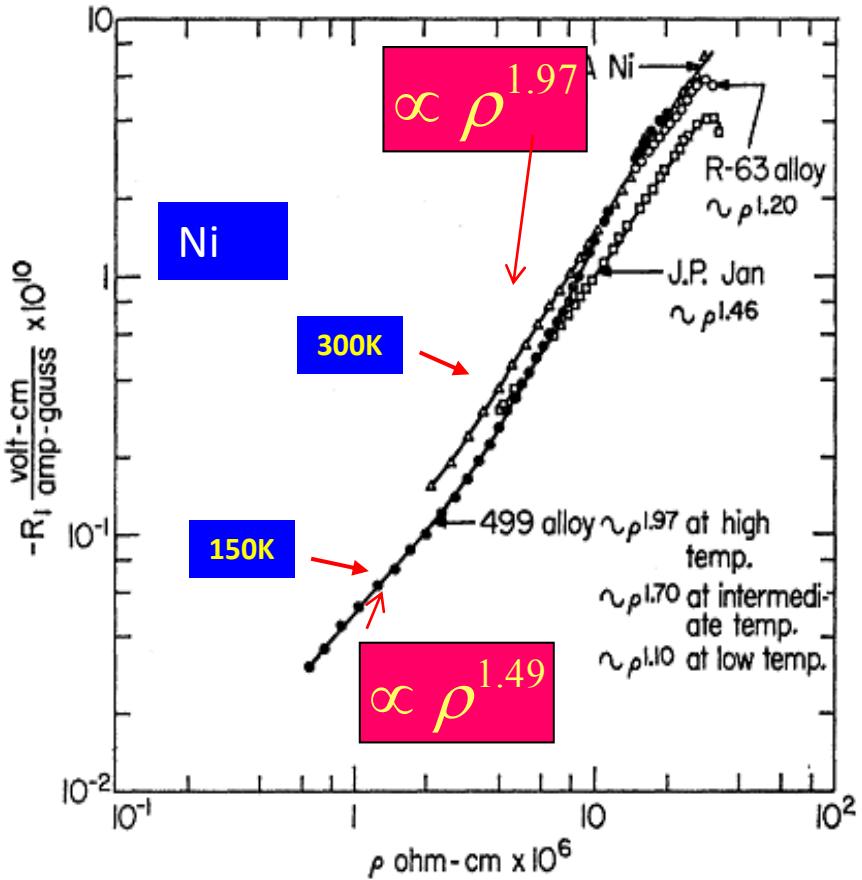
C. G. Zeng, et, al, PRL. 96, 037204 (2006)

Category 4:

$$b\rho_{xx}^\alpha$$



J. Smit, *Physica* 24, 39 (1954)



J. Mavine, *Phys. Rev.* 123, 1273 (1961)

(1) Karplus - Luttinger Intrinsic (1954)

Anomalous velocity

$$\frac{d \vec{r}(t)}{dt} = \frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}} - \vec{\Omega}_n(\vec{k}) \times \frac{d \vec{k}(t)}{dt}$$

k- space curvature

$$\frac{d \vec{k}(t)}{dt} = \frac{\partial V(\vec{r})}{\partial \vec{r}} - \vec{B}(\vec{r}) \times \frac{d \vec{r}(t)}{dt}$$

r- space curvature

G. Sundaram and Q. Niu,
Phys. Rev. B 59 (1999) 14915.

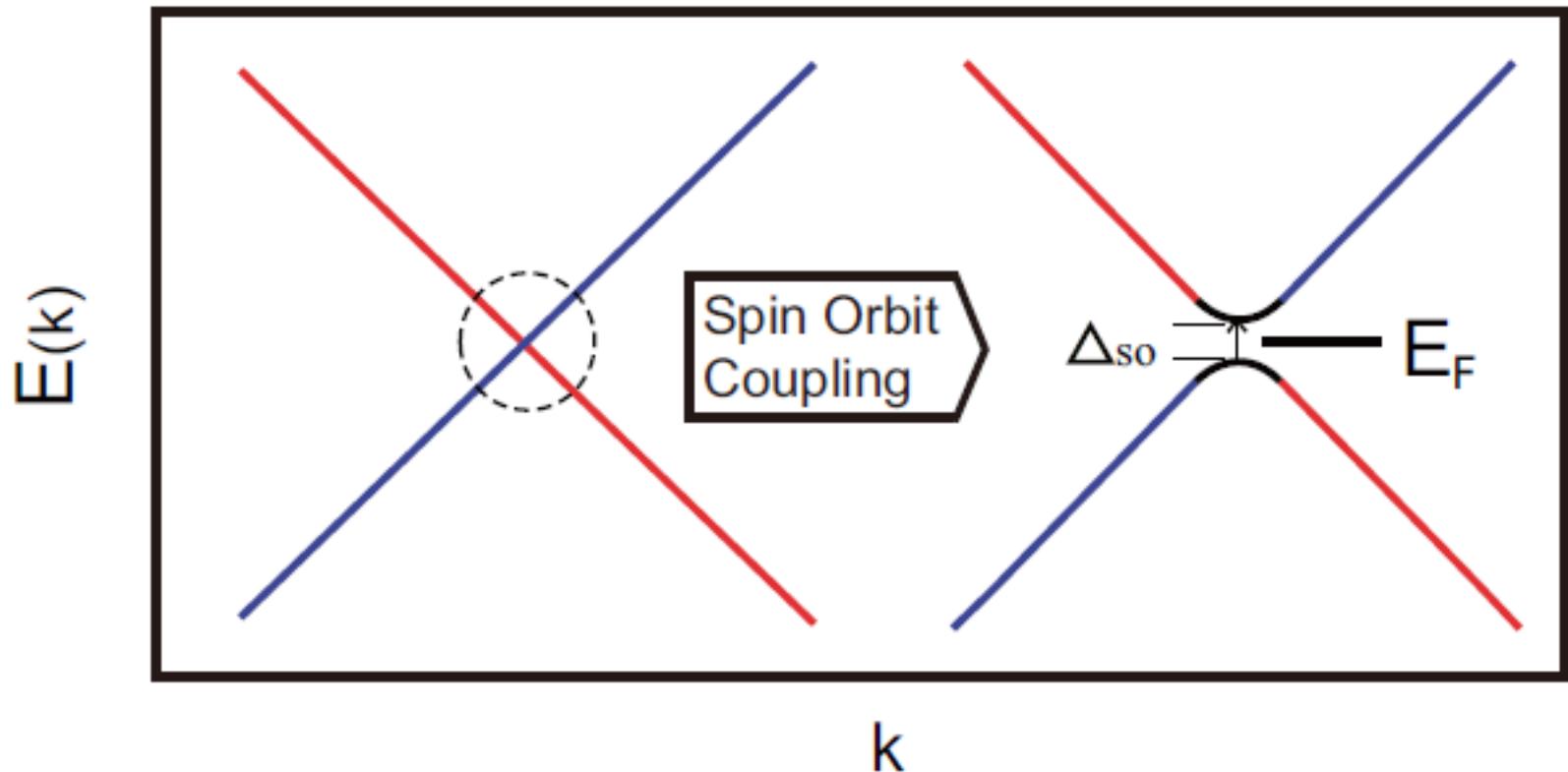
Jungwirth, Niu, MacDonald (2002), Onoda & Nagaosa (2002)

$$\sigma_{xy} = -\frac{e^2}{\hbar} \int d^3k \sum_n f(\varepsilon_n(\mathbf{k})) \boxed{\Omega_n^z(\mathbf{k})} \quad \text{Berry curvature}$$

$$\Omega_n^z(\mathbf{k}) = -\sum_{n' \neq n} \frac{2 \operatorname{Im} \langle \mathbf{k}n | v_x | \mathbf{k}n' \rangle \langle \mathbf{k}n' | v_y | \mathbf{k}n \rangle}{(\omega_{\mathbf{k}n'} - \omega_{\mathbf{k}n})^2}$$

$\sigma_{\text{int}} = \text{constant}$

$\rightarrow \rho_{int} = \sigma_{int} \rho_{xx}^2$

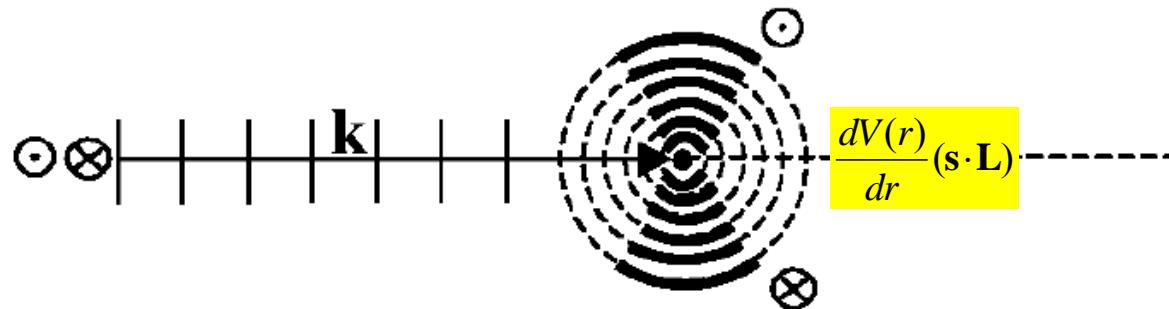


$$\Omega_n^z(\mathbf{k}) = - \sum_{n' \neq n} \frac{2 \operatorname{Im} \left\langle \mathbf{k}n \mid v_x \mid \mathbf{k}n' \right\rangle \left\langle \mathbf{k}n' \mid v_y \mid \mathbf{k}n \right\rangle}{(\omega_{\mathbf{k}n'} - \omega_{\mathbf{k}n})^2}$$

e.g., S. Onoda, N. Sugimoto, N. Nagaosa, PRL, 97, 126602 (2006)

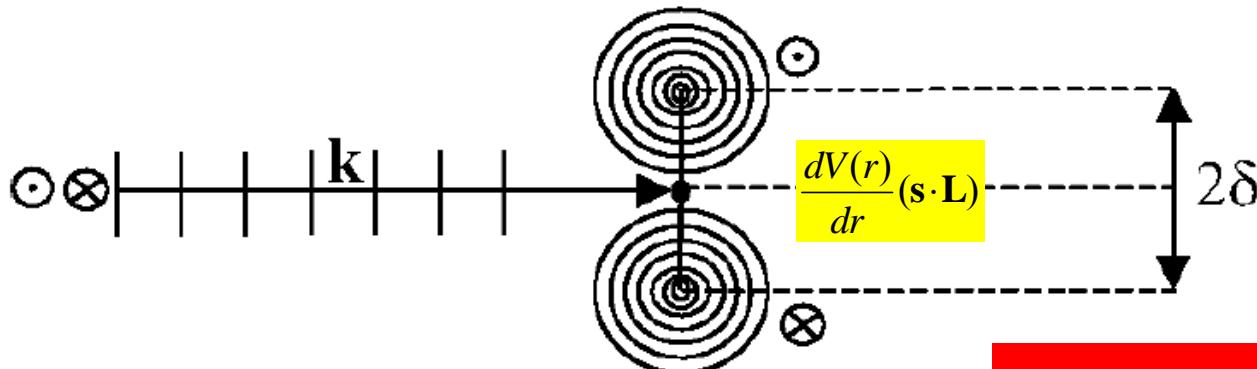
(2) Skew-scattering (Smit, 1955)

$$\rho_{AH} \propto \rho_{xx}$$



(3) Side-jump (Berger, 1970)

$$\rho_{AH} \propto \rho_{xx}^2$$



$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

Question 1: Intrinsic and Extrinsic ?

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

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PHYSICAL REVIEW LETTERS

30 AUGUST 1999

Spin Hall Effect

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Department of Physics, University of California, San Diego, La Jolla, California 92093-0319

(Received 24 February 1999)

a variety of mechanisms have been proposed to explain the origin of the coefficient R_s . These include skew scattering by impurities and phonons, and the “side jump” mechanism [1]. In early work it was also proposed that the effect will arise in the absence of periodicity-breaking perturbations [2], but this is generally believed not to be correct [1].

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

PHYSICAL REVIEW B, VOLUME 64, 014416

Theory of the anomalous Hall effect from the Kubo formula and the Dirac equation

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(Received 24 January 2001; published 13 June 2001)

It is now accepted²² that two mechanisms are responsible for the anomalous Hall effect: the skew-scattering proposed by Smit¹⁴ and the side-jump proposed by Berger.¹⁶

Anomalous Hall effect There is a term in the Hall resistivity of a ferromagnet when the field is applied in the z -direction, perpendicular to the plane of the film, in addition to the normal Hall effect (3.53). This is the anomalous Hall effect, which varies with the magnitude of the magnetization M :

$$\varrho_{xy} = \mu_0(R_h H' + R_s M). \quad (5.83)$$

The anomalous Hall effect is yet another consequence of spin-orbit coupling. The symmetry of the radial component of the Lorentz force $\mathbf{j} \times \mathbf{B}$ which produces the normal Hall effect is the same as the symmetry of the spin-orbit interaction $\mathbf{L} \cdot \mathbf{S}$ since $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, $\mathbf{p} \propto \mathbf{j}$, $\mathbf{S} \propto \mu_0 \mathbf{M}$.

In a ferromagnet the anomalous Hall effect varies as the macroscopic average magnetization. Generally ϱ_{xy} vary as ϱ_{xx} and as ϱ_{xx}^2 , which are associated with mechanisms. Deviation of the electron trajectories due to spin-orbit interaction is known as skew scattering.

Writing $\varrho_m = \mu_0 R H'$, the Hall angle ϕ_H is defined as ϱ_m / ϱ_{xx} . Thus $\phi_H = \alpha + \beta \varrho_{xx}$; α is the angle. The second term is often larger. It is associated with mechanism due to impurity scattering. If $\delta \approx 0.1$ nm is the side jump, the Hall angle here is δ / λ , which is proportional to ϱ_{xx} . Here λ is the mean free path.

Anomalous Hall effect

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

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and Cross-Correlated Materials Research Group (CMRG), and Correlated Electron
Research Group (CERG), ASI, RIKEN, Wako, 351-0198 Saitama, Japan*

Jairo Sinova

*Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA
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Shigeki Onoda

Condensed Matter Theory Laboratory, ASI, RIKEN, Wako, 351-0198 Saitama, Japan

A. H. MacDonald

Department of Physics, University of Texas at Austin, Austin, Texas 78712-1081, USA

N. P. Ong

Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

(Published 13 May 2010)

On the theoretical front, the adoption of the Berry-phase

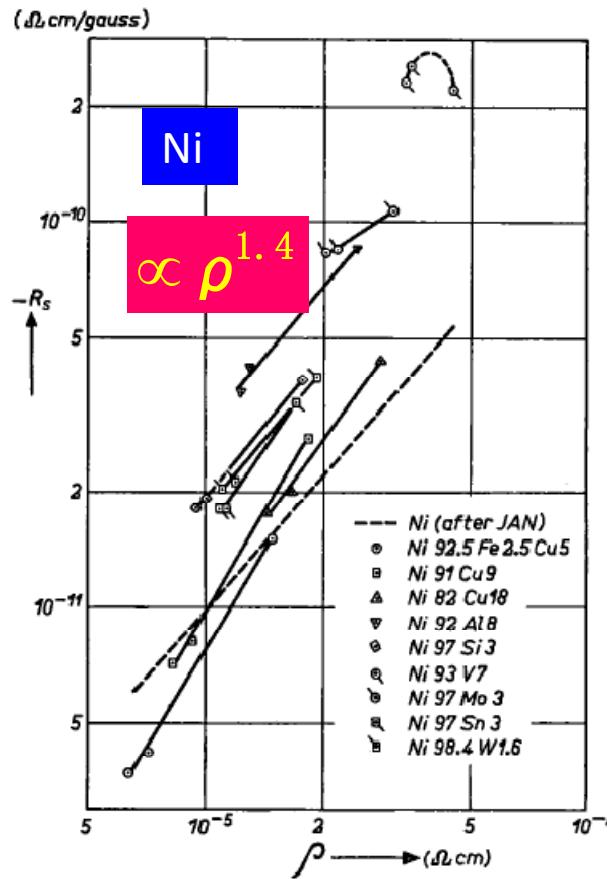
concepts has established a link between the AHE and the topological nature of the Hall currents. On the experimental front, new experimental studies of the AHE in transition metals, transition-metal oxides, spinels, pyrochlores, and metallic dilute magnetic semiconductors have established systematic trends. These two developments, in concert with first-principles electronic structure calculations, strongly favor the dominance of an intrinsic Berry-phase-related AHE mechanism in metallic ferromagnets with moderate conductivity. The intrinsic AHE can be expressed in terms of the Berry-phase curvatures and it is therefore an intrinsic quantum-mechanical property of a perfect crystal.

Question 2:

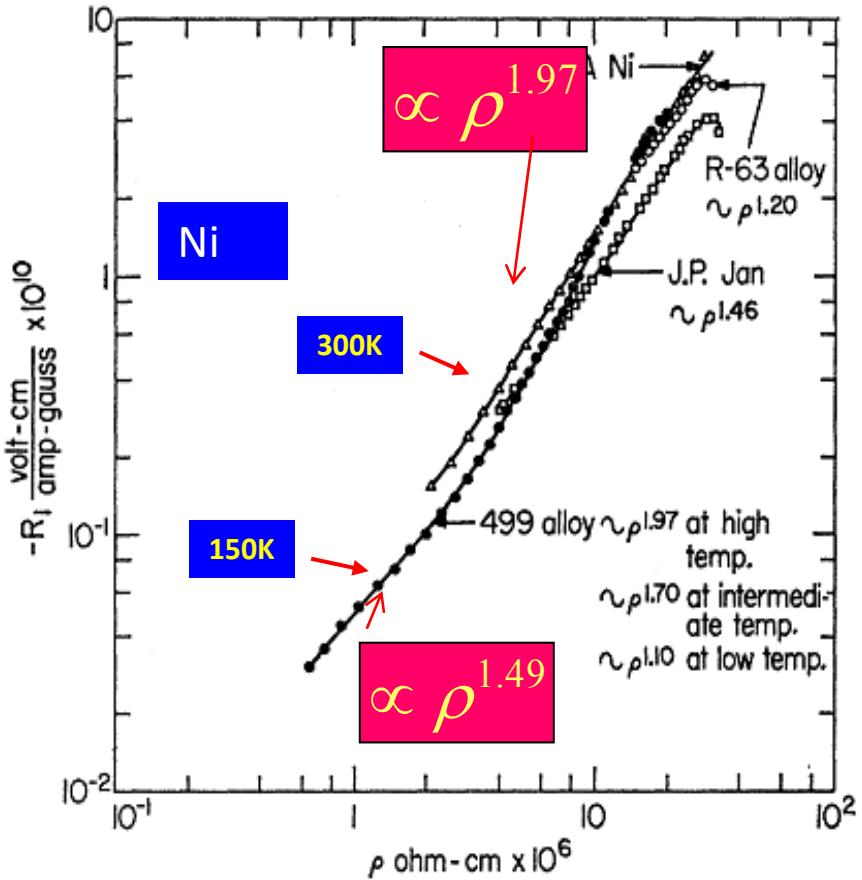
Temperature Dependence in the AHE-Ni ?

Category 4:

$$b\rho_{xx}^\alpha$$



J. Smit, *Physica* 24, 39 (1954)



J. Mavine, *Phys. Rev.* 123, 1273 (1961)

Transition metals

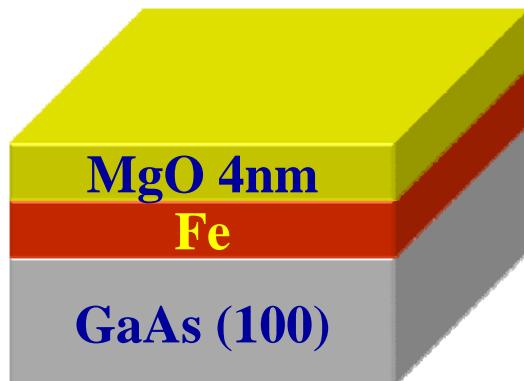
Anomalous Hall Conductivity ($\Omega\text{cm})^{-1}$

	Theory	Exp
bcc Fe	752	1032
hcp Co	477	480
fcc Ni	-2203	-646

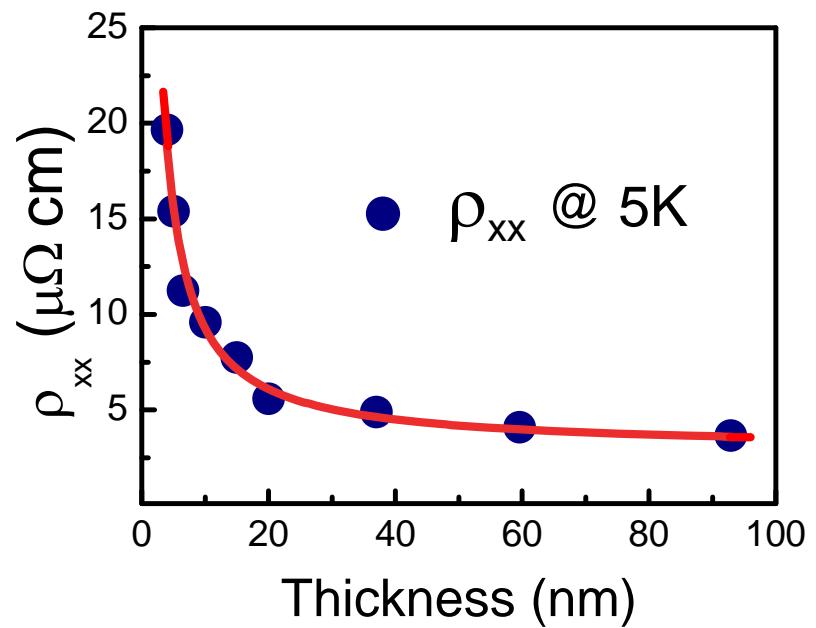
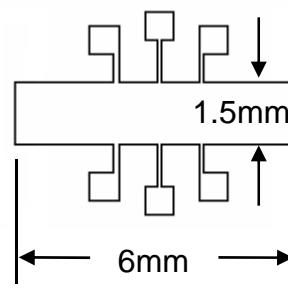
X. J. Wang, et.al. *Phys. Rev. B* 76, 195109 (2007)

2. Experimental Results

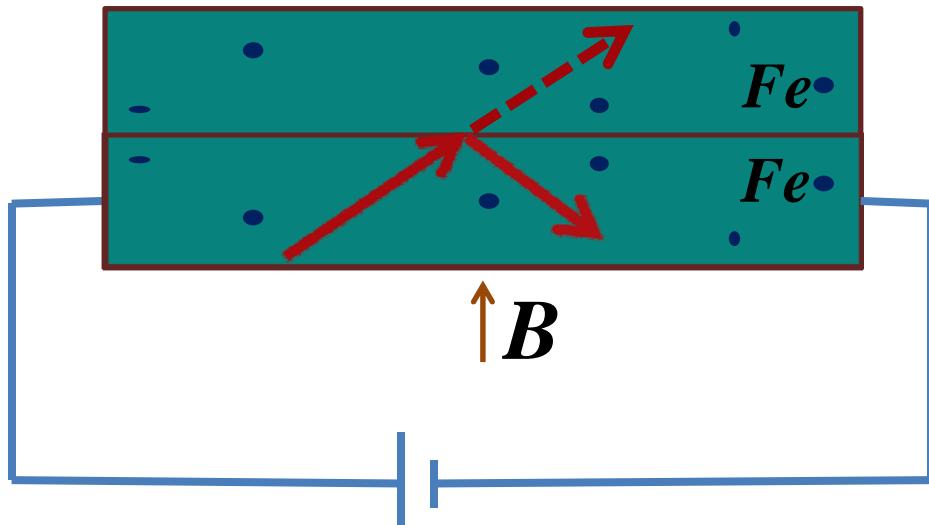
$$\rho_{ah} = f(\rho_{xx})$$



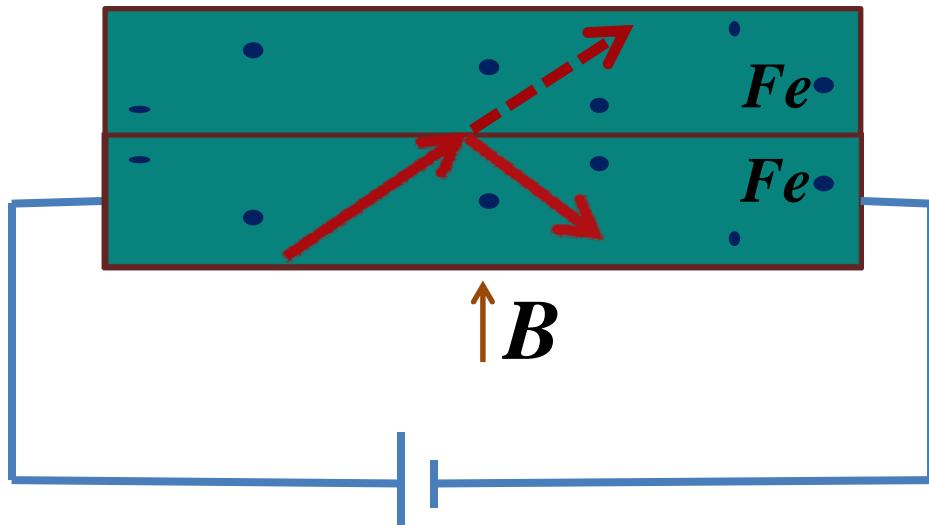
Tuning with thickness and temperature



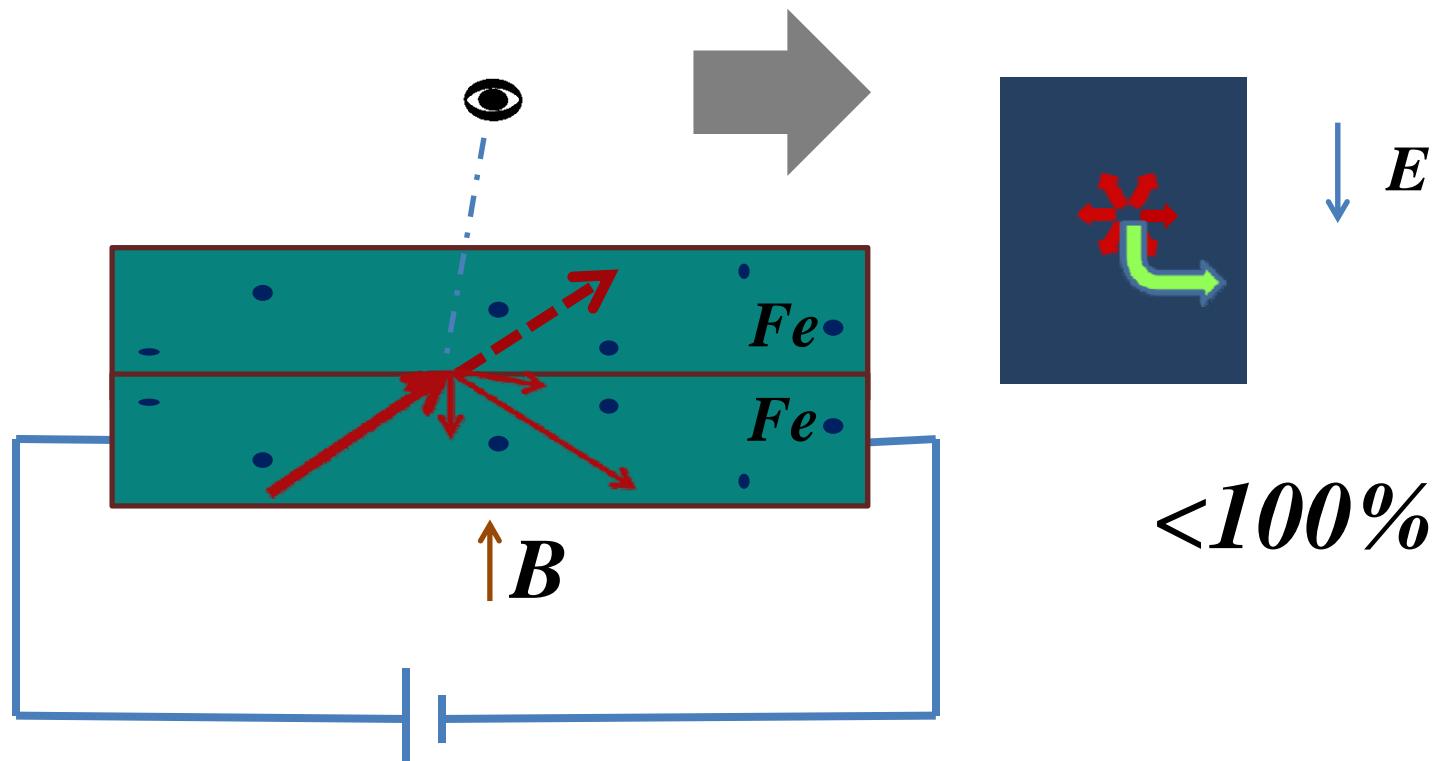
- bcc single crystal iron
- Current flow along [110]



100%



100%



<100%

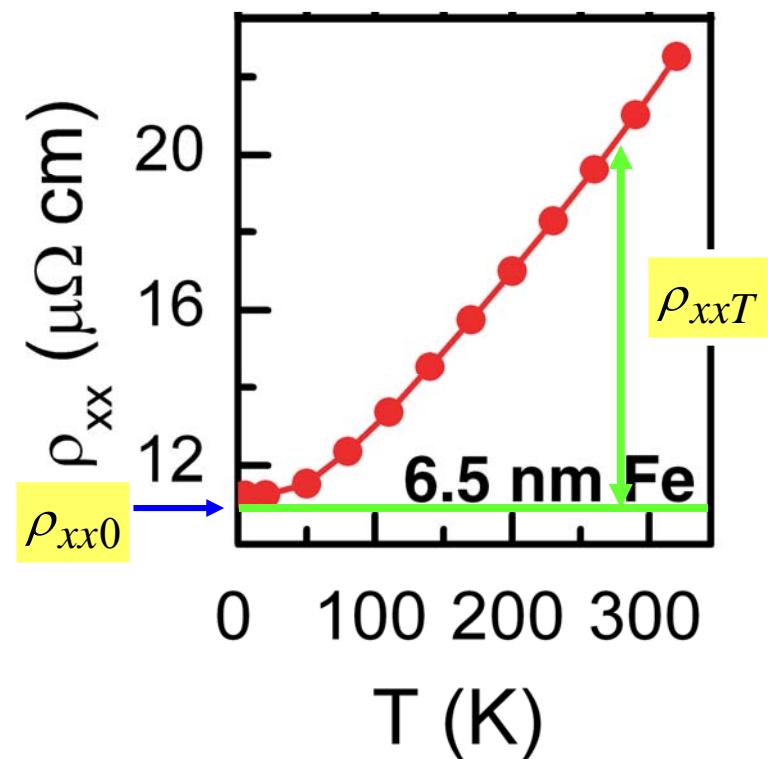
Old scaling:

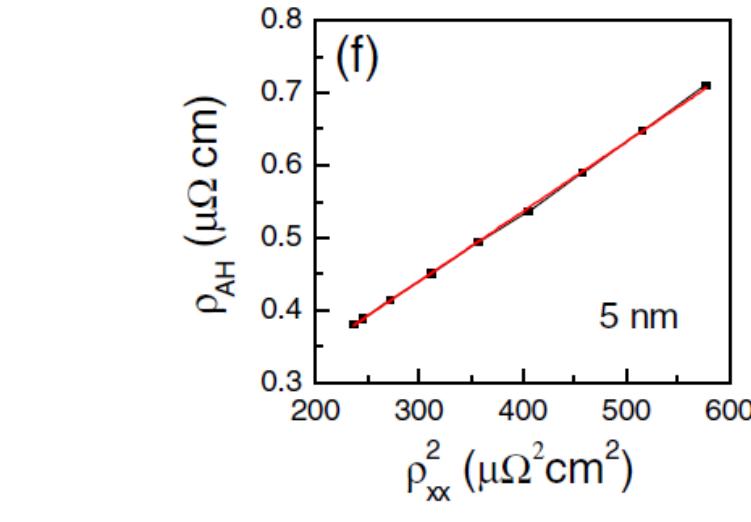
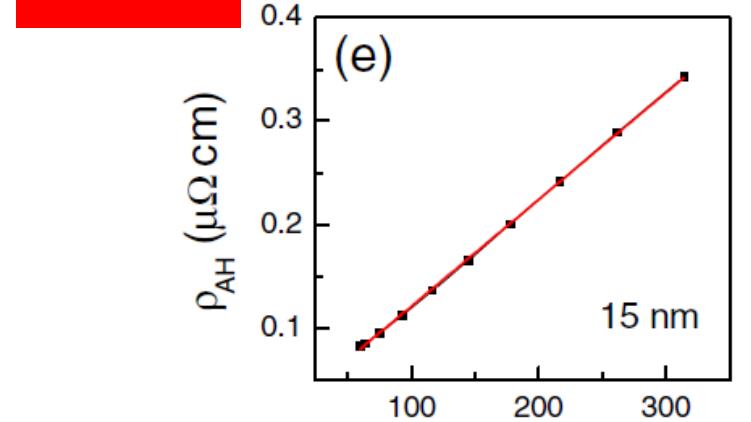
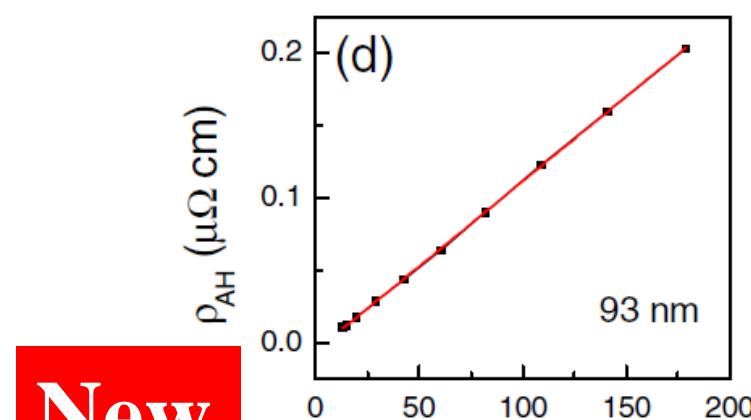
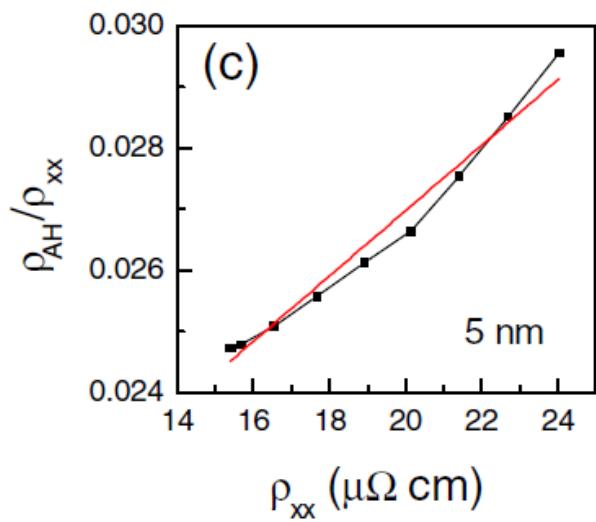
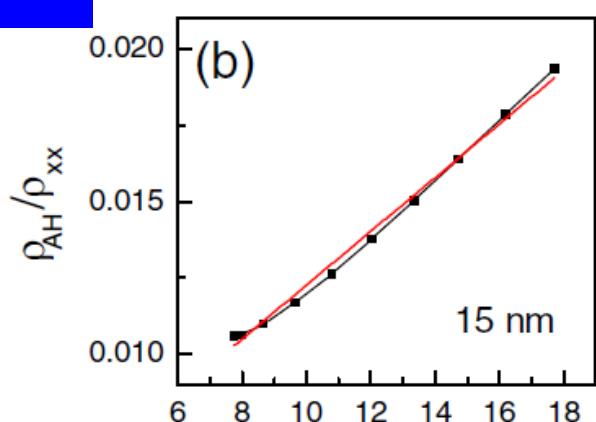
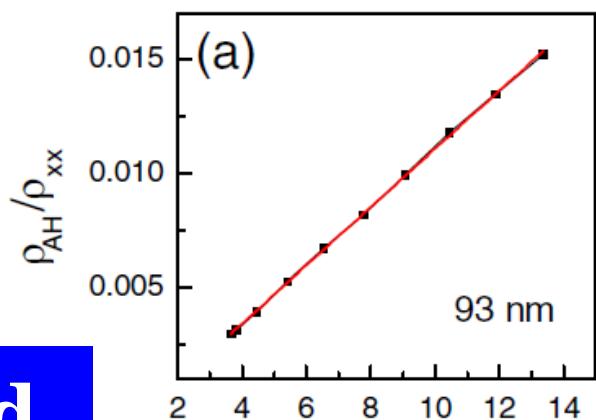
$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

$$\frac{\rho_{ah}}{\rho_{xx}} = a + b\rho_{xx}$$

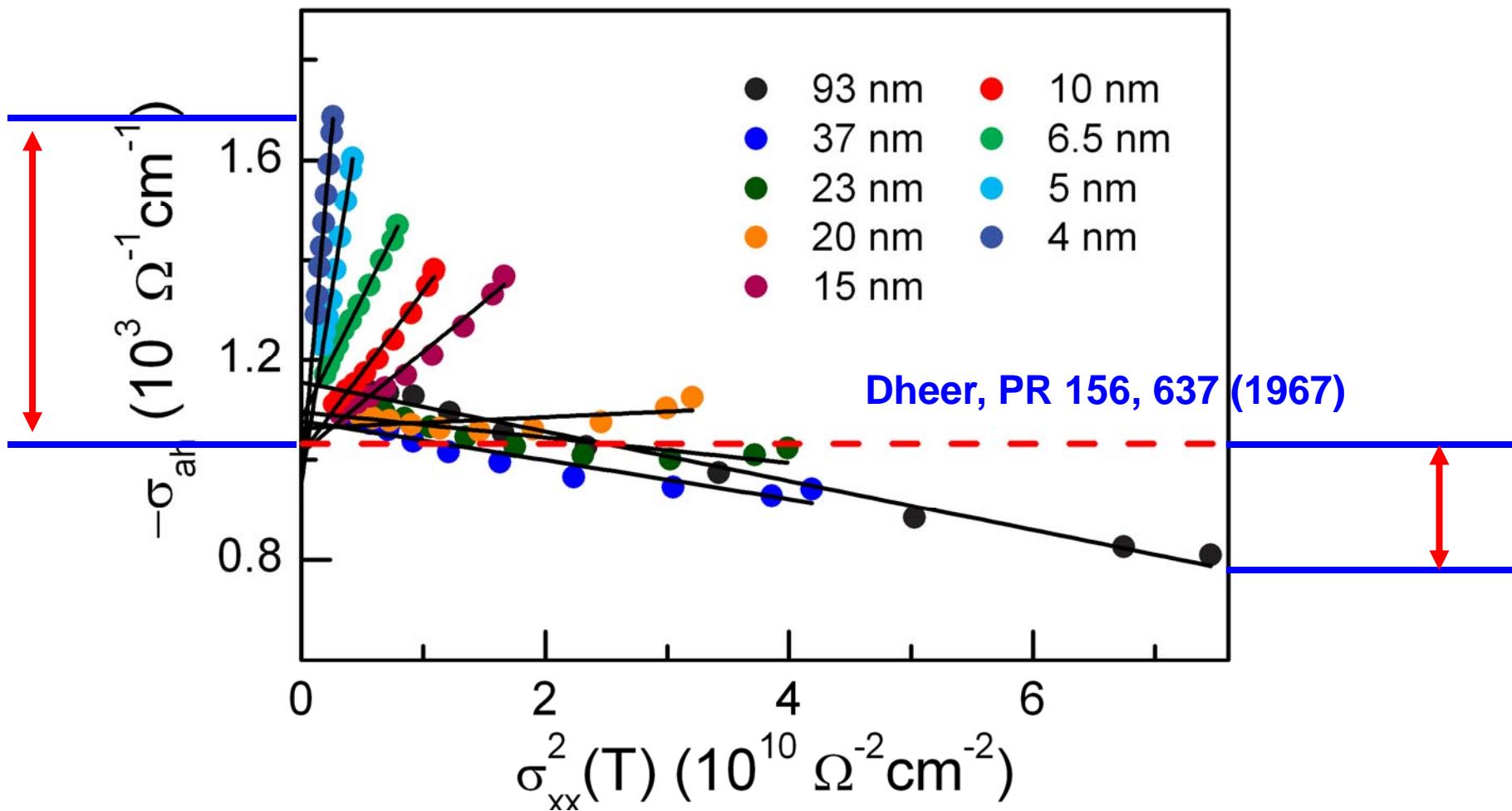
New scaling:

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$



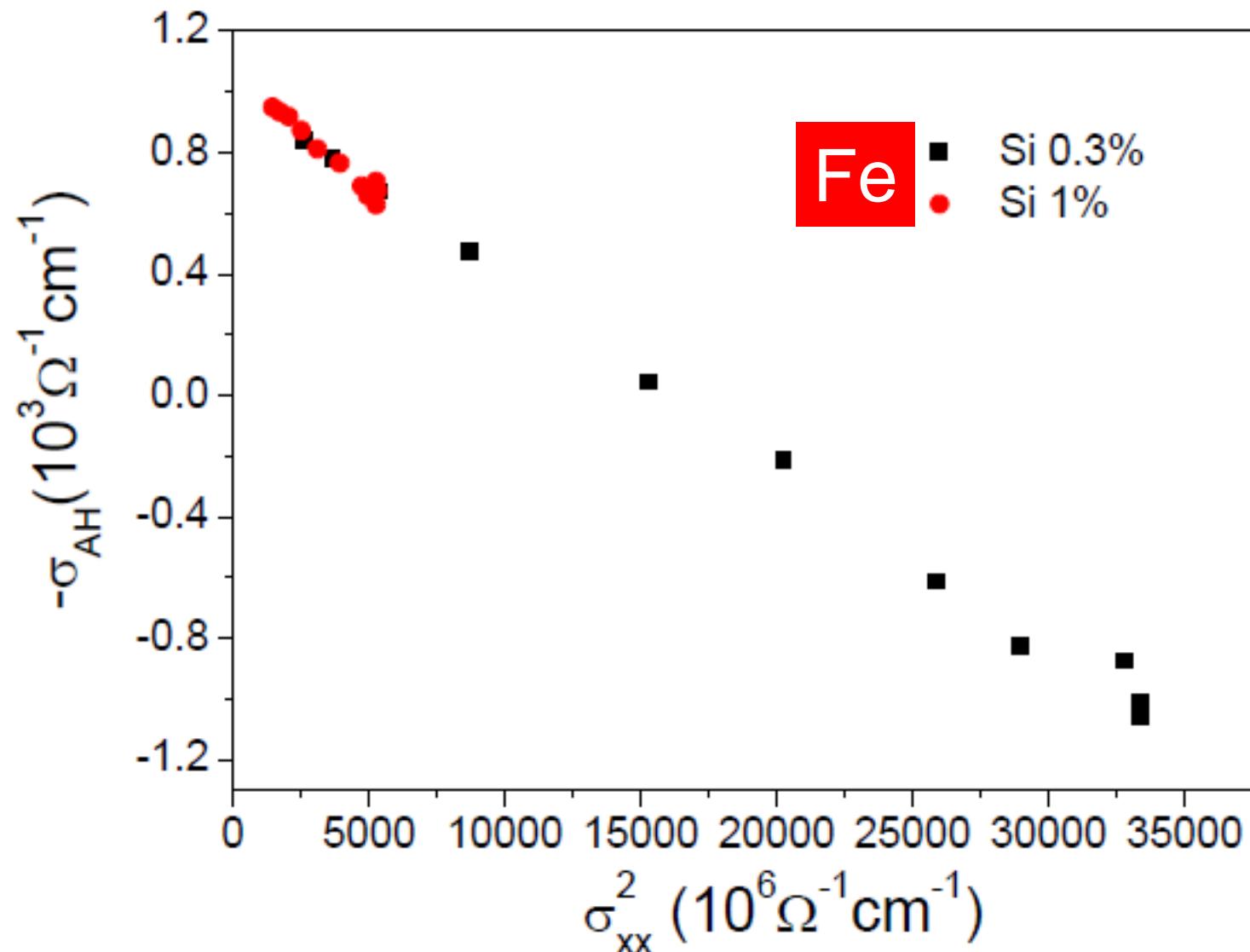


$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$



$$b \approx 1.1 \times 10^3 \Omega^{-1} \text{cm}^{-1} = -\sigma_{int}$$

Intercept: **0.3%: $0.99 \times 10^3 \Omega^{-1} \text{cm}^{-1}$, 1%: $1.07 \times 10^3 \Omega^{-1} \text{cm}^{-1}$**



First Principles Calculation of Anomalous Hall Conductivity in Ferromagnetic bcc Fe

Yugui Yao,^{1,2}

$$-\sigma_{\text{int}}^{\text{Theory}} \approx 0.75 \times 10^3 \quad \Omega^{-1} \text{cm}^{-1}$$

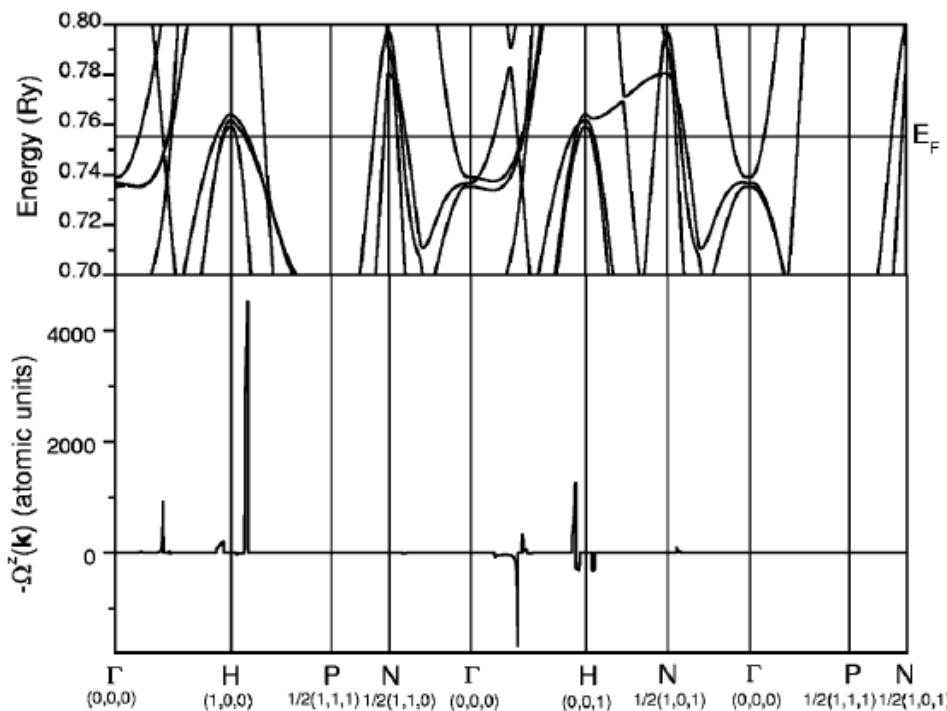
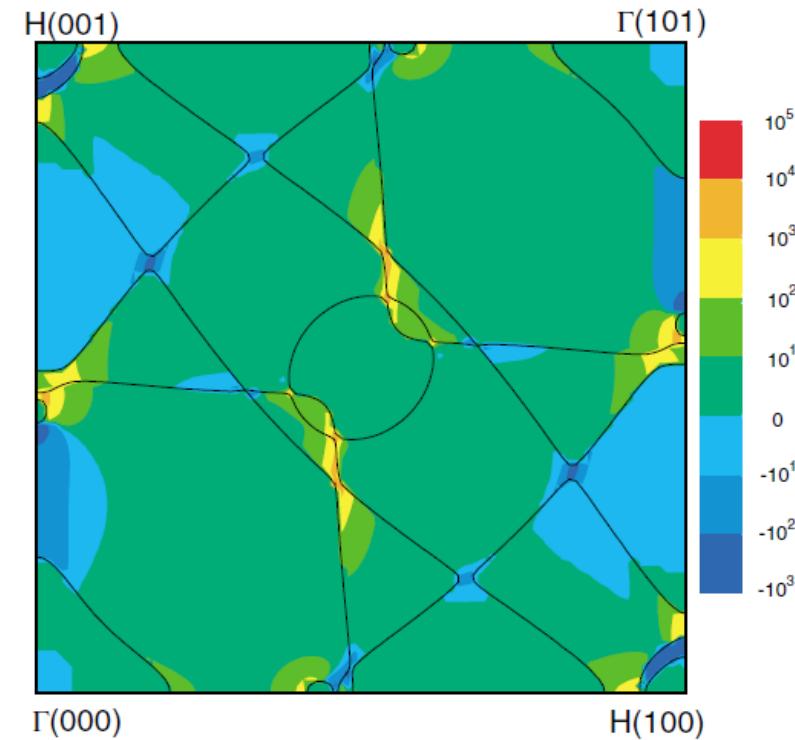
²Inter

$$-\sigma_{\text{int}}^{\text{Experiment}} \approx 1.1 \times 10^3 \quad \Omega^{-1} \text{cm}^{-1}$$

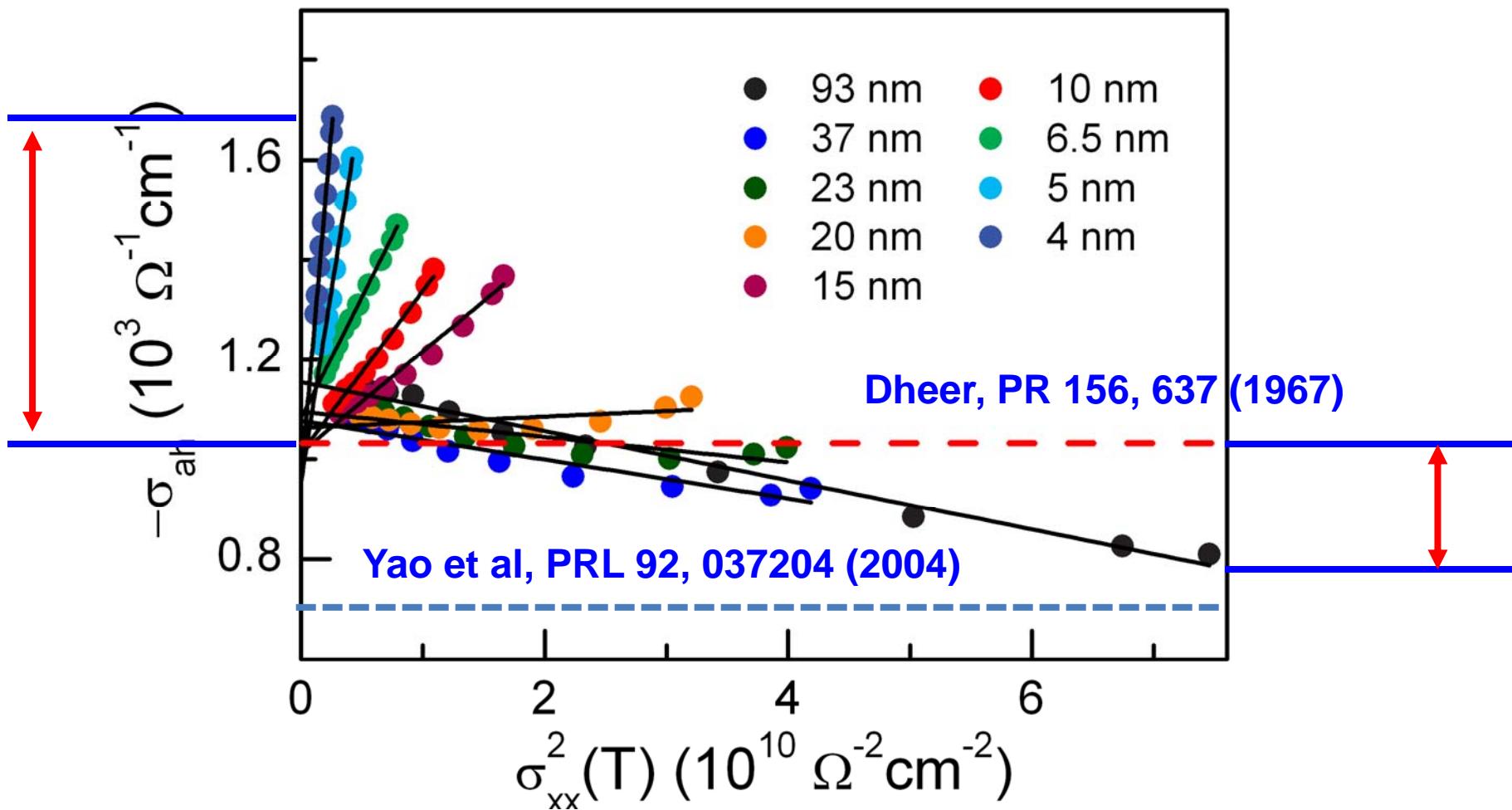
Heng Wang,³

Hina

(Received 14 July 2003; published 22 January 2004)

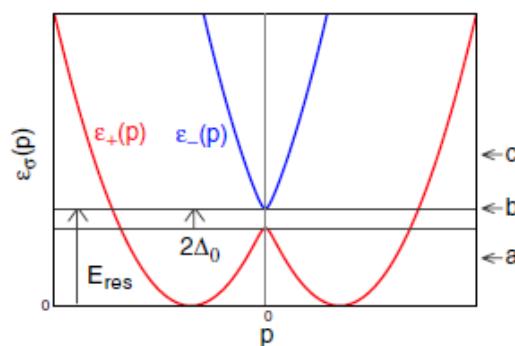
FIG. 2. Band structure near Fermi energy (upper panel) and Berry curvature $\Omega^z(\mathbf{k})$ (lower panel) along symmetry lines.FIG. 3 (color). Fermi surface in (010) plane (solid lines) and Berry curvature $-\Omega^z(\mathbf{k})$ in atomic units (color map).

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

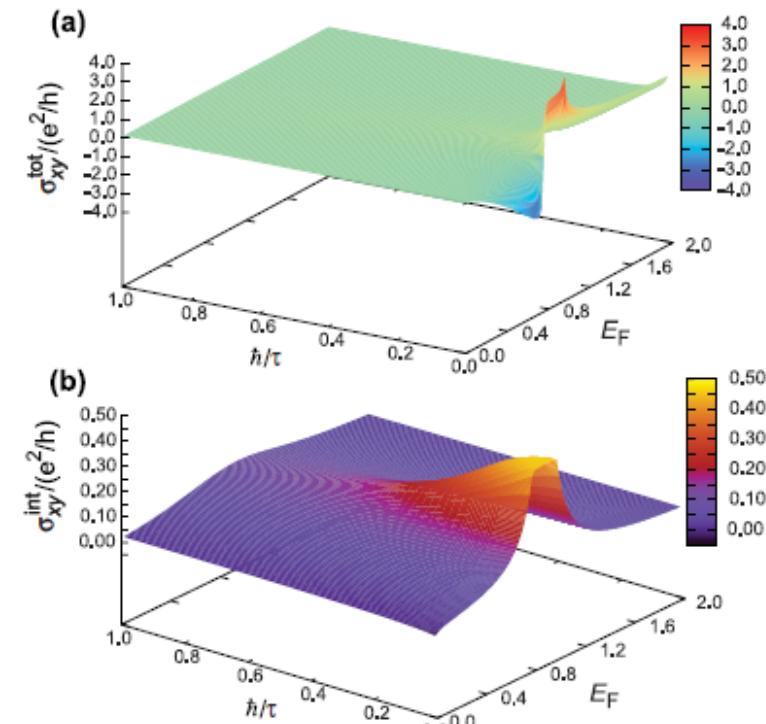


$$b \approx 1.1 \times 10^3 \Omega^{-1} cm^{-1} = -\sigma_{int}$$

Intrinsic Versus Extrinsic Anomalous Hall Effect in Ferromagnets

Shigeki Onoda,^{1,*} Naoyuki Sugimoto,² and Naoto Nagaosa^{2,3}¹*Spin Superstructure Project, ERATO, Japan Science and Technology Agency,
c/o Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan*²*CREST, Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan*³*Correlated Electron Research Center, National Institute of Advanced Industrial Science and Technology, Tsukuba 305-8562, Japan*
(Received 23 May 2006; published 18 September 2006)FIG. 1 (color online). Energy dispersions for \hat{H}_0 in Eq. (2).

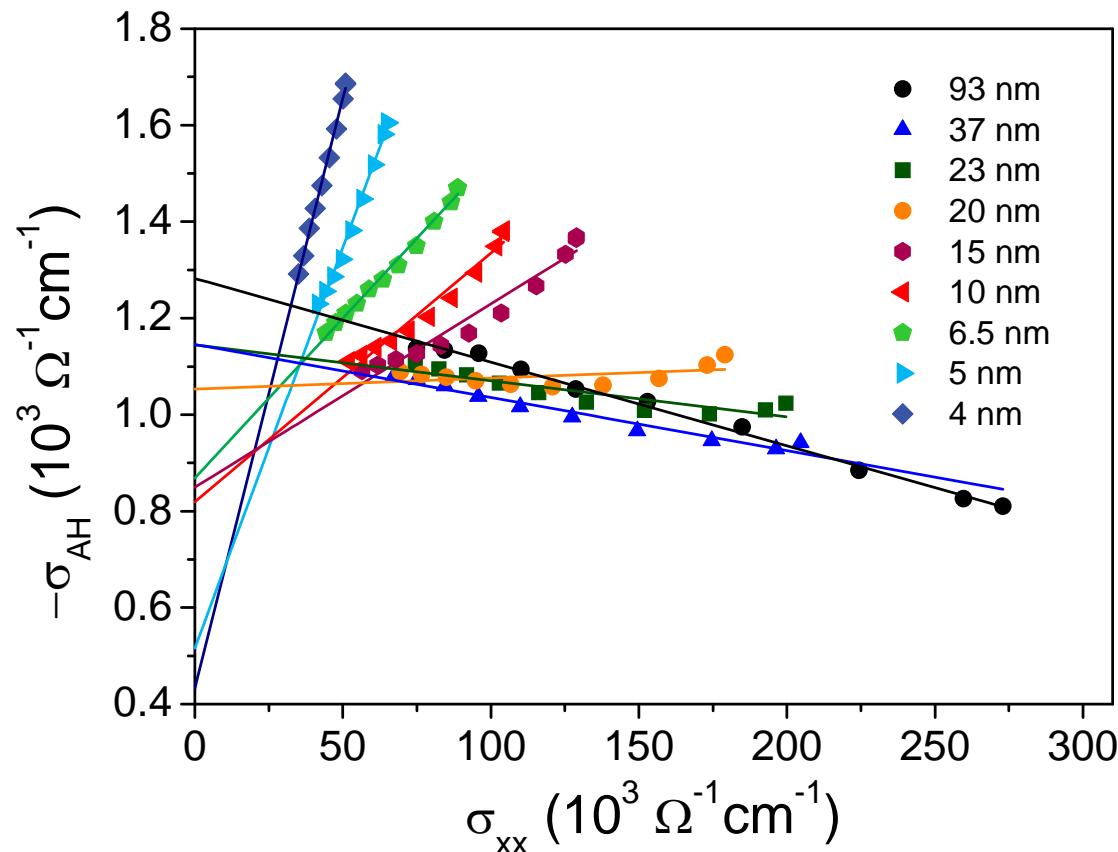
$$\begin{aligned} \hat{H}_0 + \hat{H}_{\text{imp}} = & -\Delta_0 \hat{\sigma}^z + \lambda \mathbf{p} \cdot \hat{\boldsymbol{\sigma}} \times \mathbf{e}^z + \frac{\mathbf{p}^2}{2m} \\ & + v_{\text{imp}} \sum_{\mathbf{r}_{\text{imp}}} \delta(\mathbf{r} - \mathbf{r}_{\text{imp}}), \end{aligned} \quad (2)$$

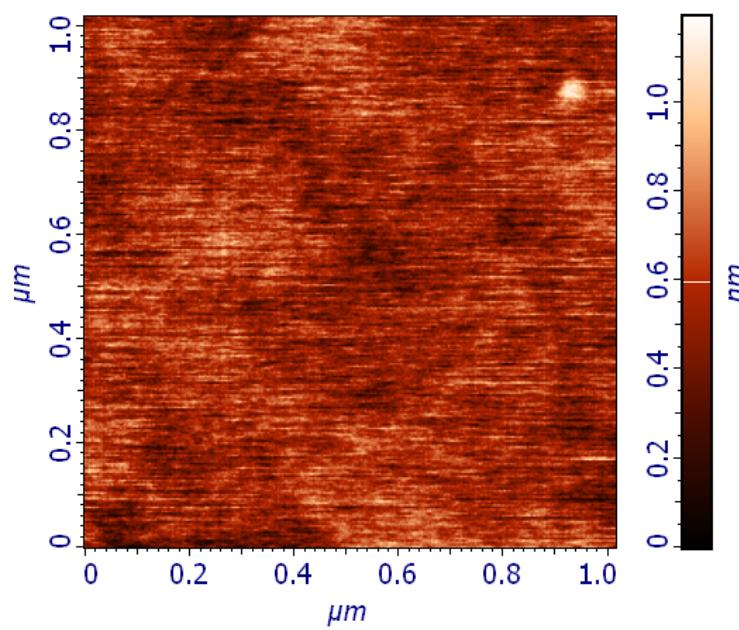
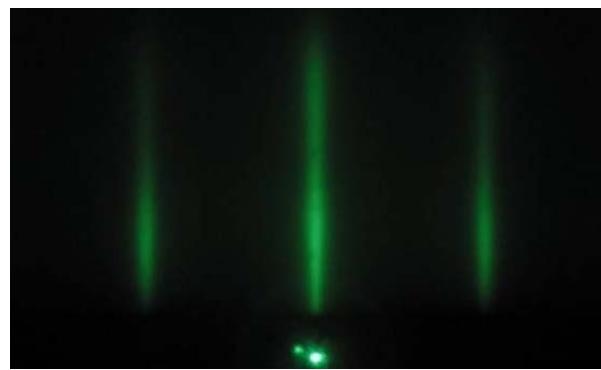
FIG. 2 (color). (a) The total anomalous Hall conductivity σ_{xy}^{tot} against E_F and \hbar/τ in an energy unit of $E_{\text{res}} = 1.0$. (b) The intrinsic contribution σ_{xy}^{int} for the same parameter values. Note the difference of the scales for σ_{xy} in (a) and (b).

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

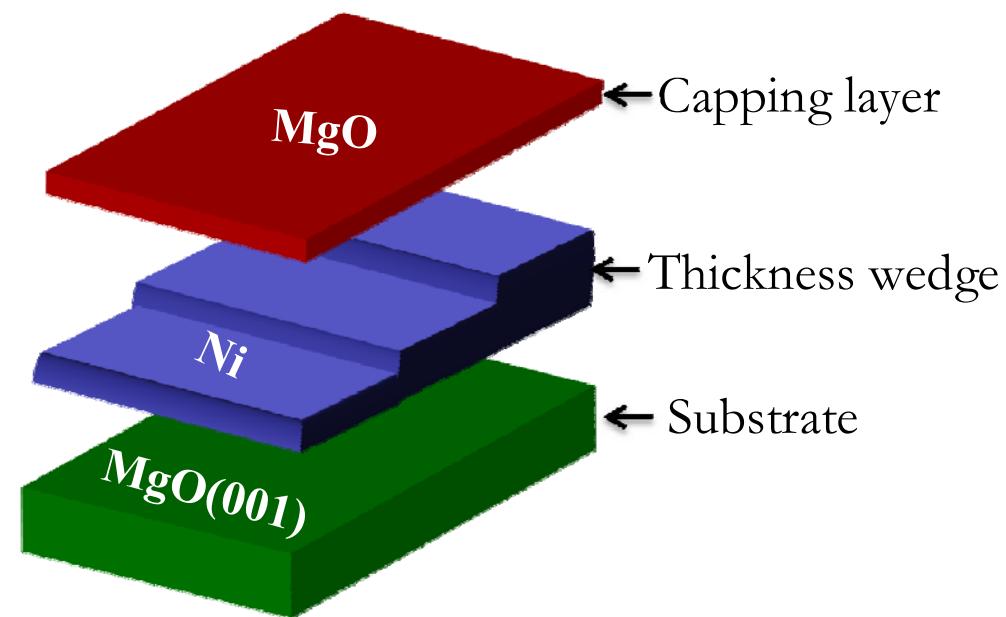
Old scaling:

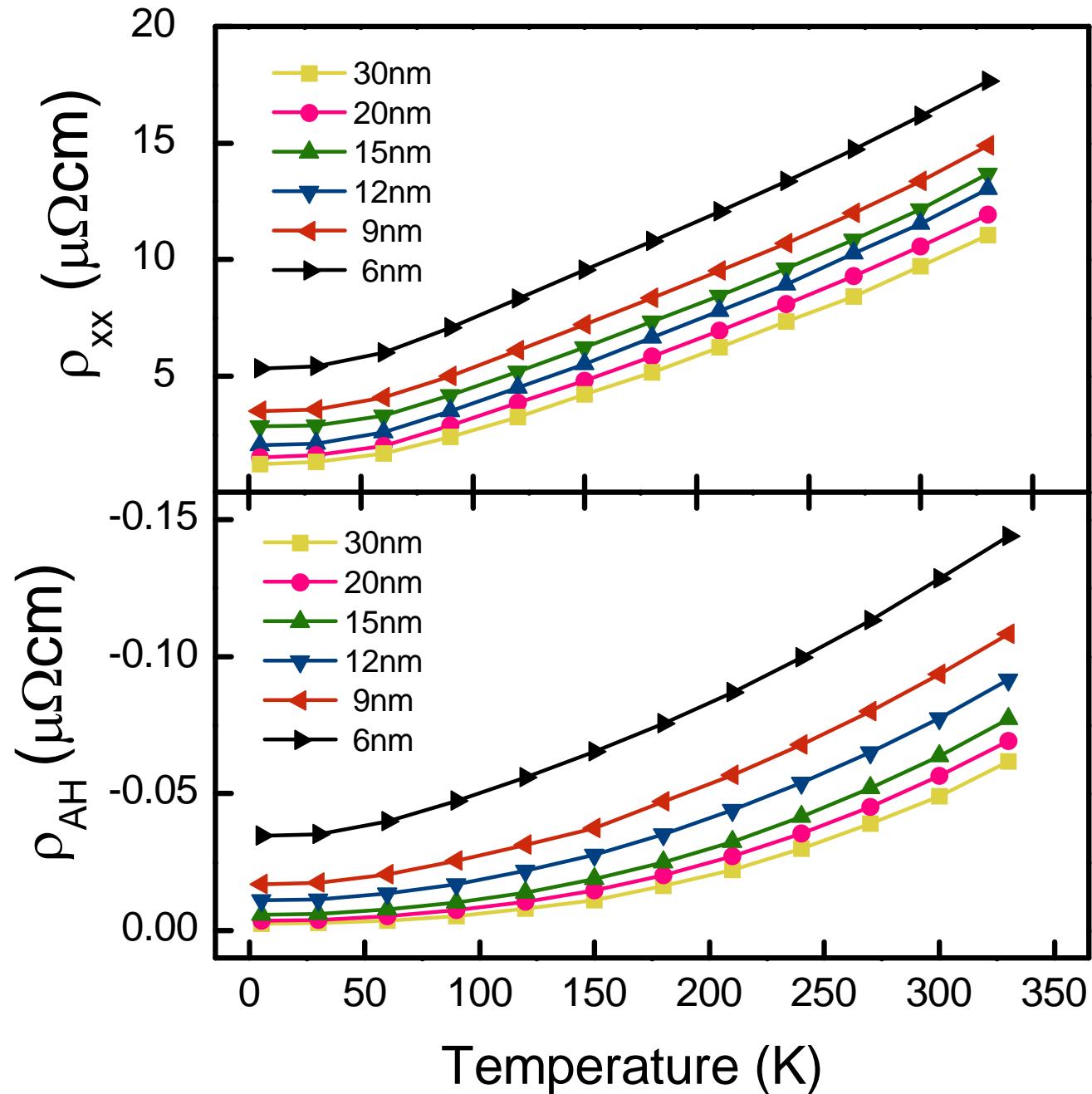
$$\sigma_{ah} = -a\sigma_{xx} - b$$

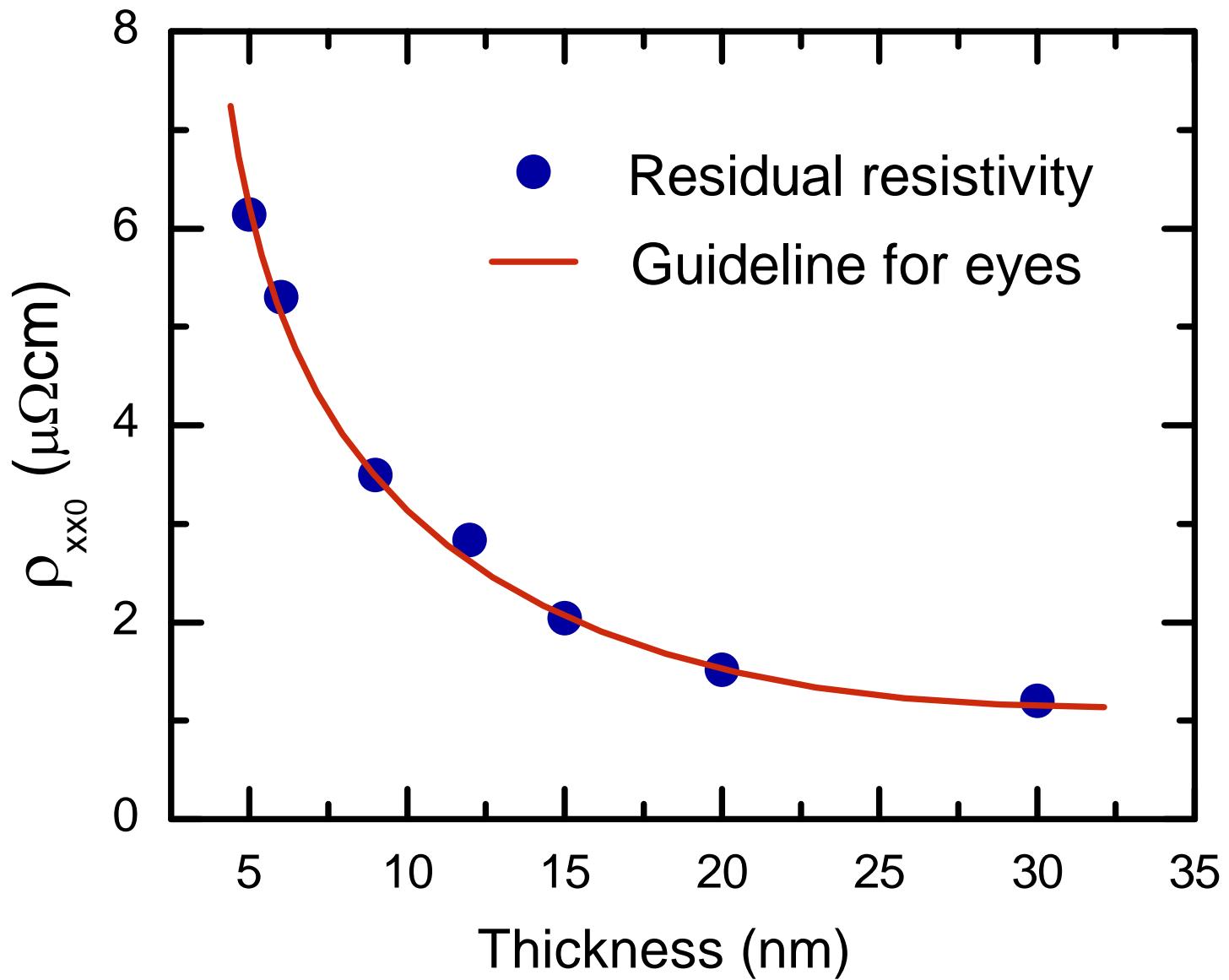


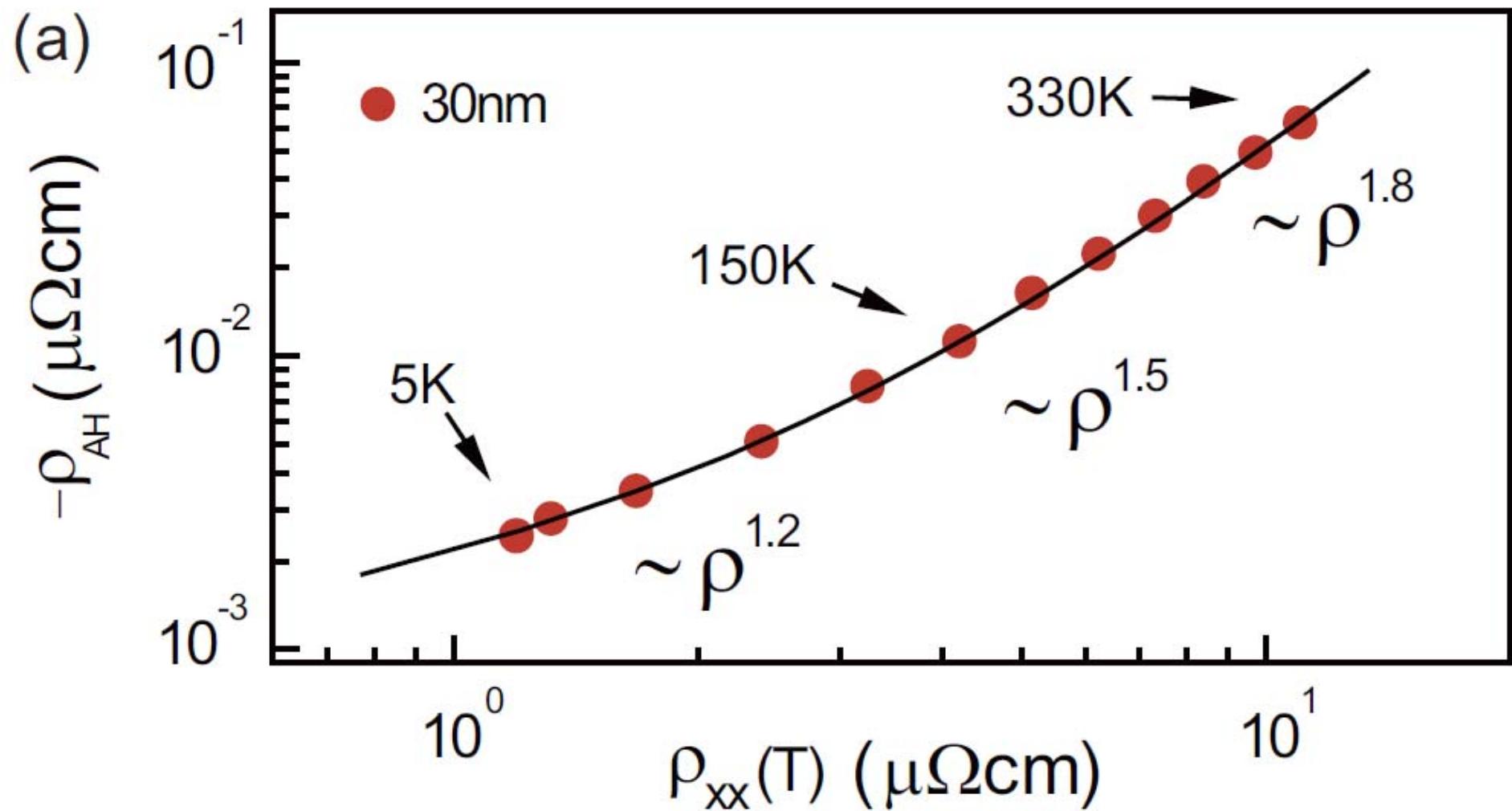


Roughness<0.2nm

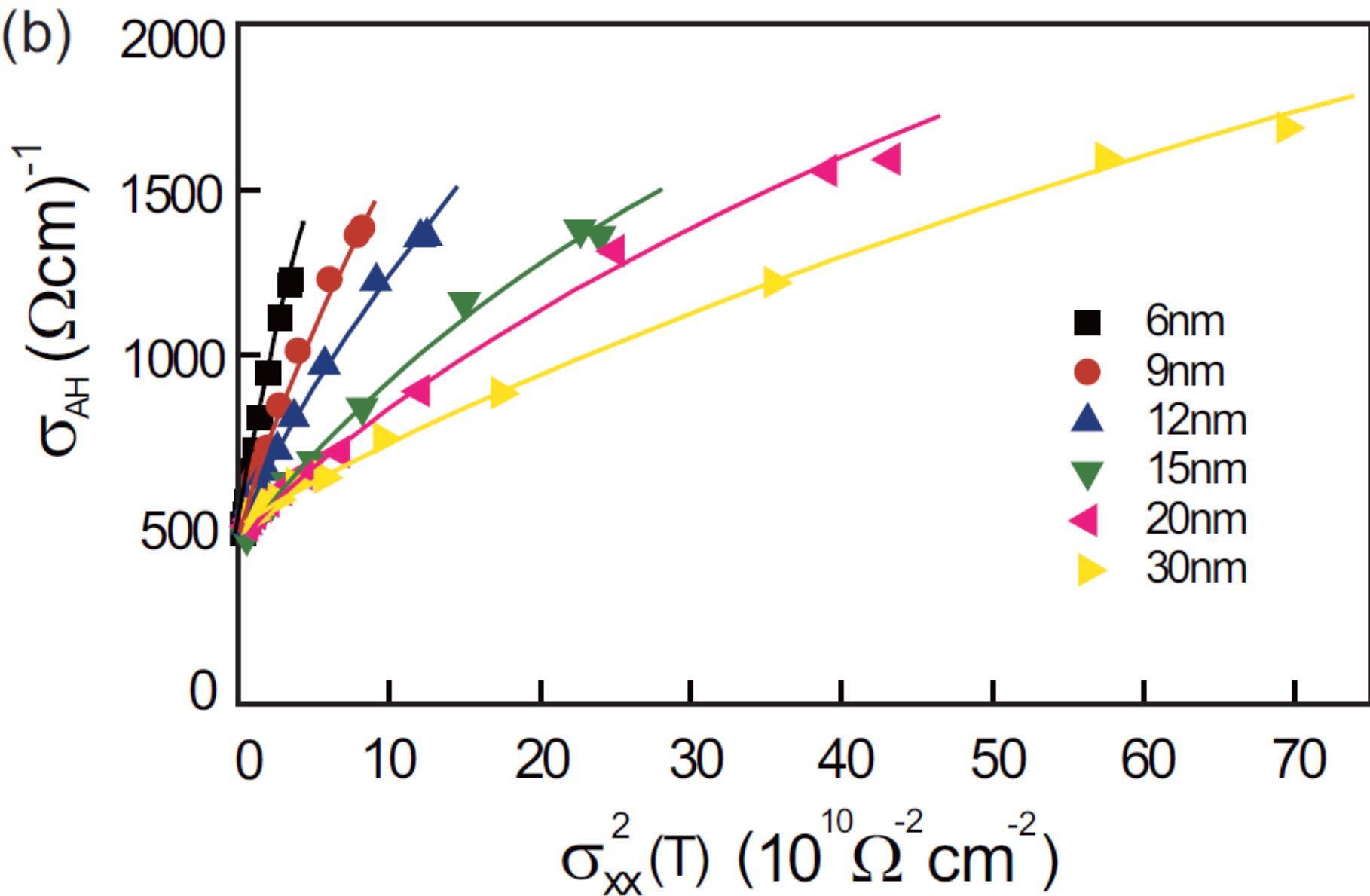






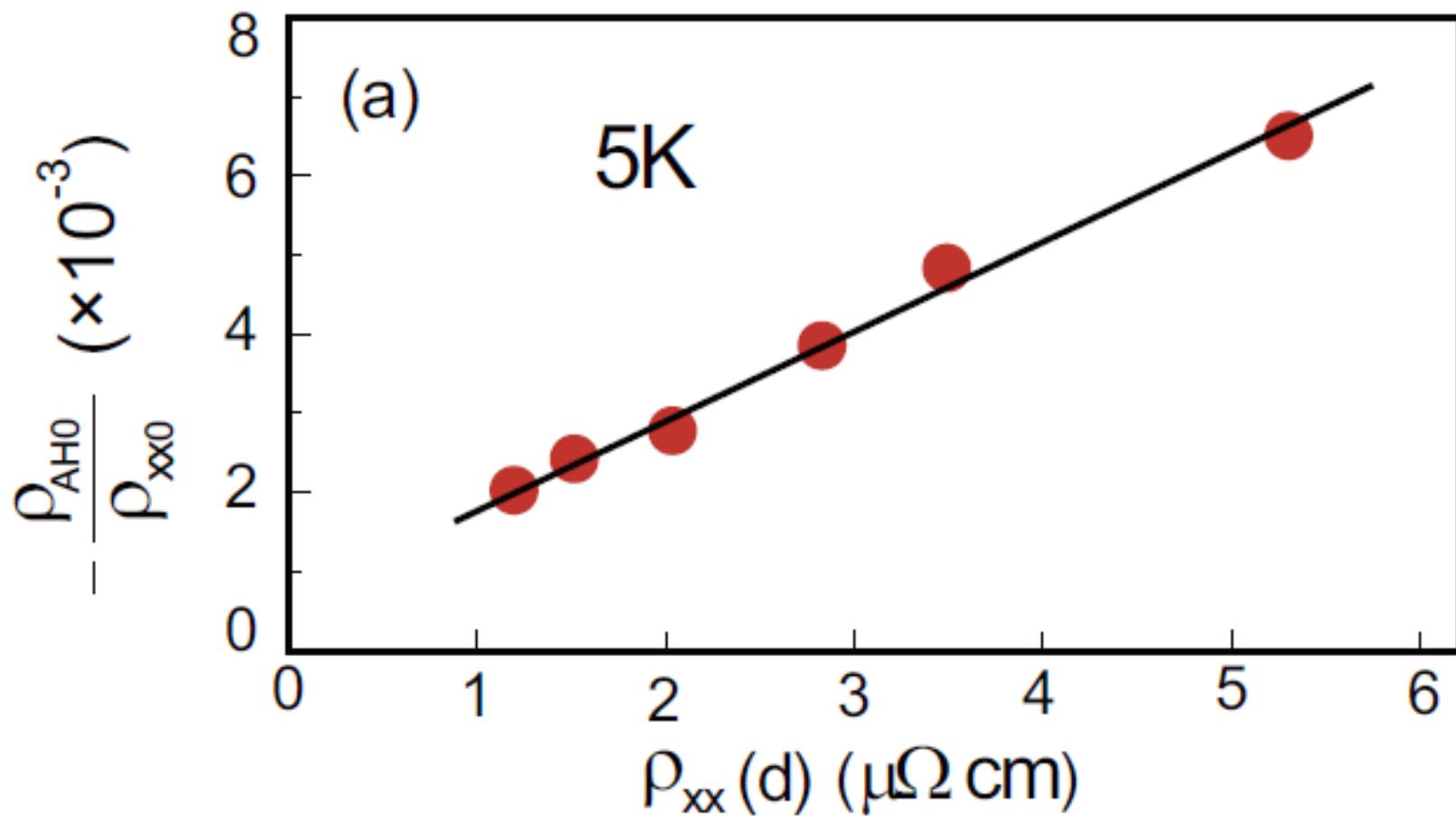


$$\sigma_{ah} = \sigma(\sigma_{xx0}, \sigma_{xx}) = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$



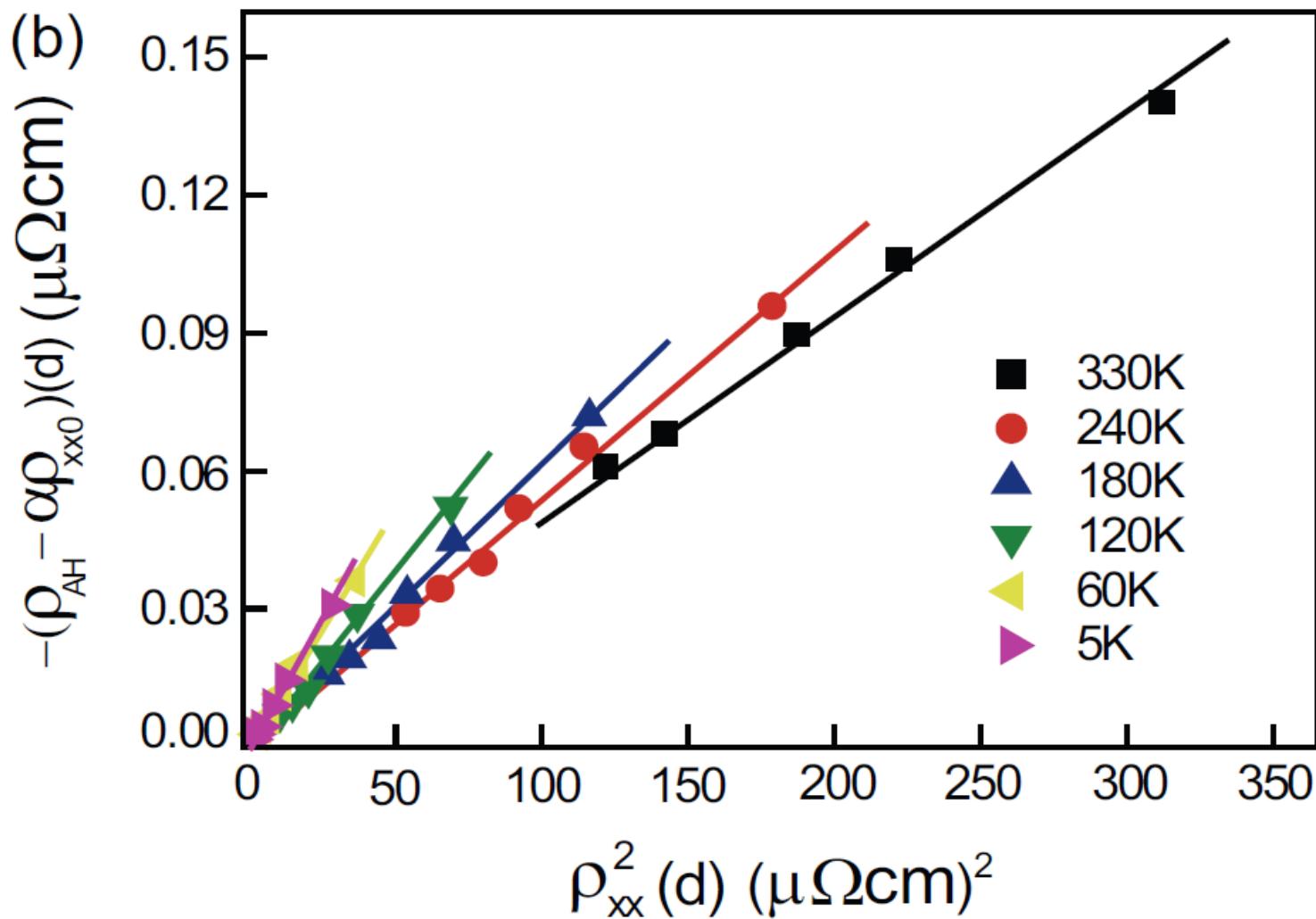
$$\rho_{ah} = f(\rho_{xx0}, \rho_{xx}) = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b \rho_{xx}^2$$

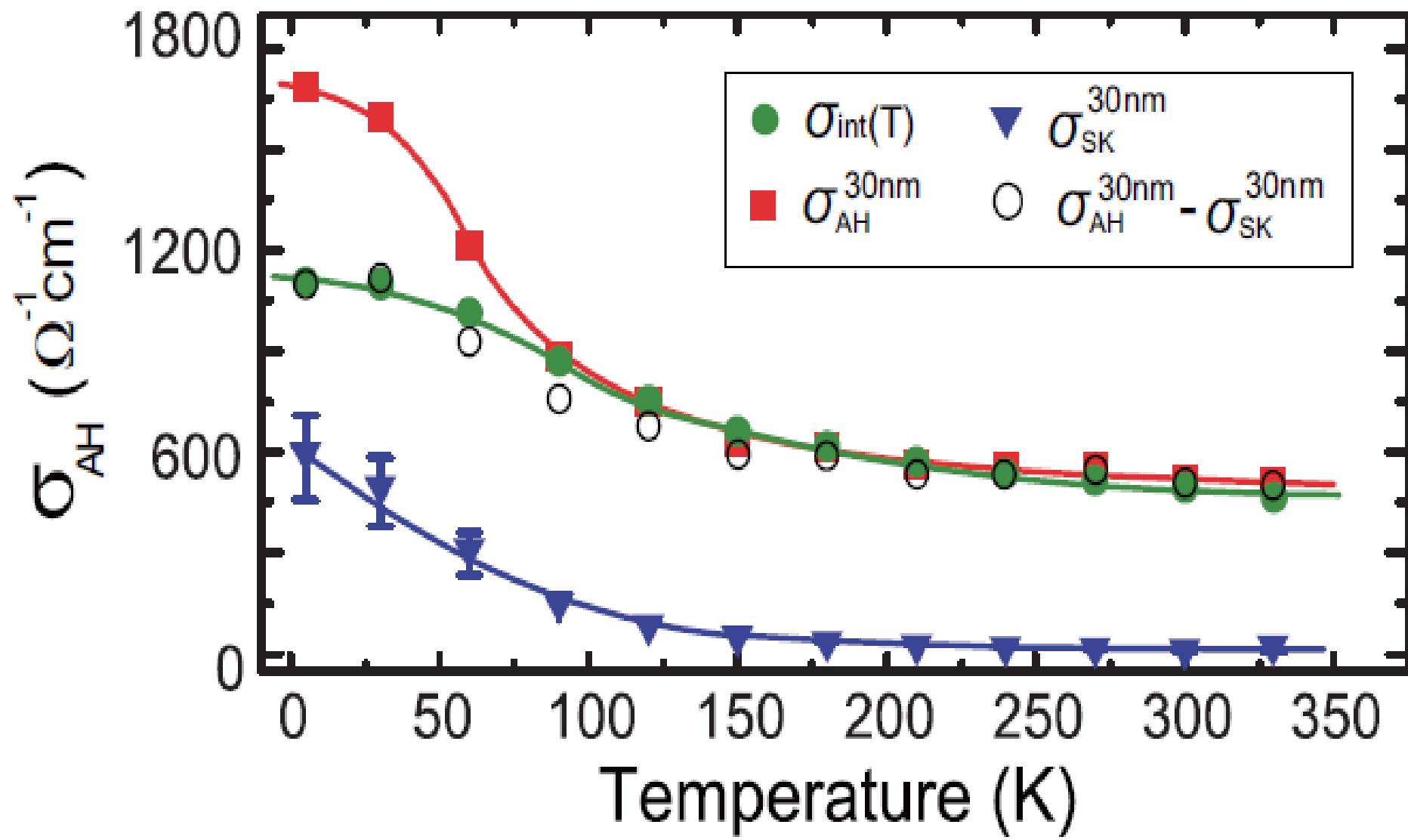
$$\rho_{ah} = \alpha \rho_{xx0} + (\beta + b) \rho_{xx0}^2$$



$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b \rho_{xx}^2$$

$$\rho_{ah} - \alpha \rho_{xx0} = \beta \rho_{xx0}^2 + b(T) \rho_{xx}^2$$

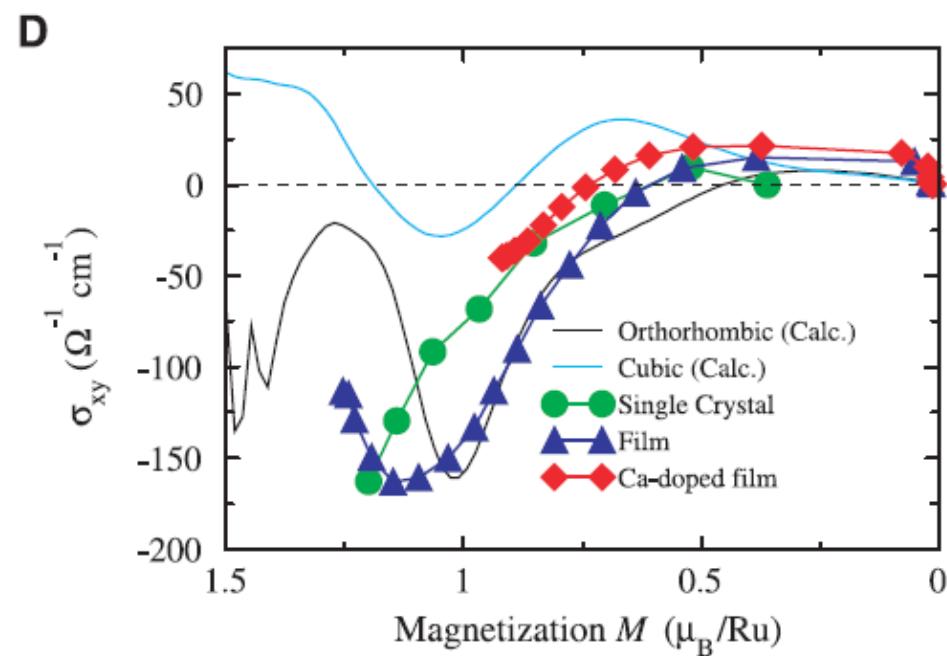
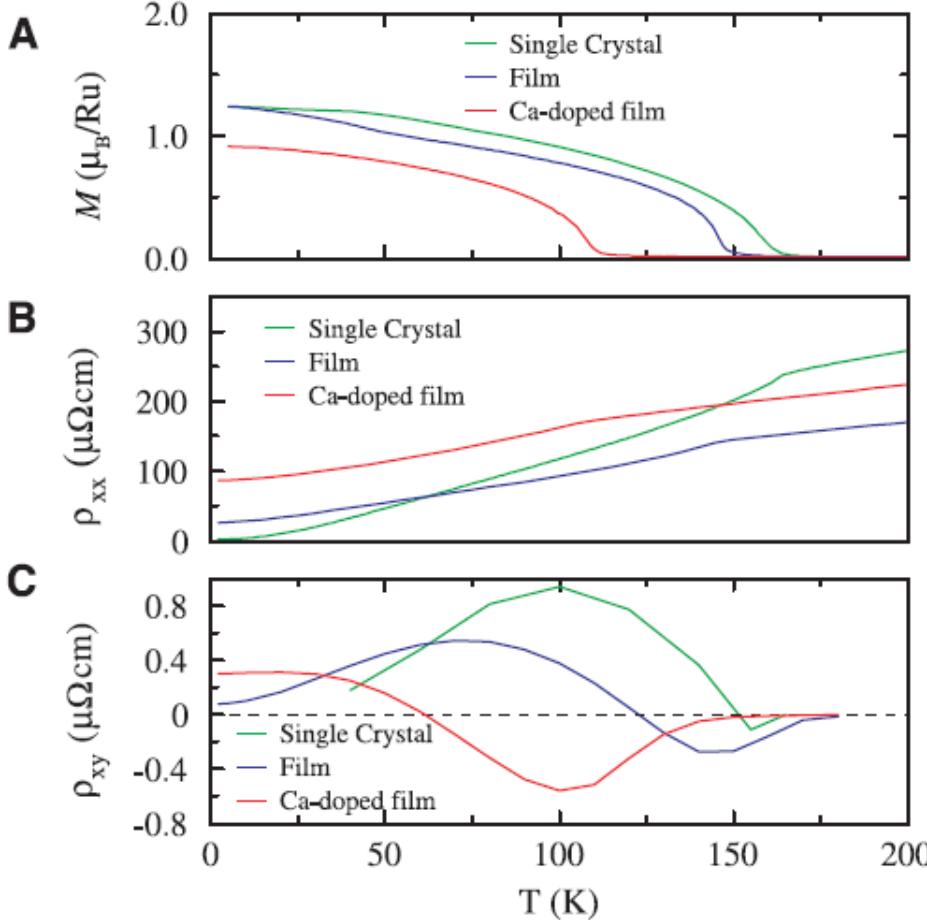




The Anomalous Hall Effect and Magnetic Monopoles in Momentum Space

3 OCTOBER 2003 VOL 302 SCIENCE

Zhong Fang,^{1,2*} Naoto Nagaosa,^{1,3,4} Kei S. Takahashi,⁵
Atsushi Asamitsu,^{1,6} Roland Mathieu,¹ Takeshi Ogasawara,³
Hiroyuki Yamada,³ Masashi Kawasaki,^{3,7} Yoshinori Tokura,^{1,3,4}
Kiyoyuki Terakura⁸



Linear Magnetization Dependence of the Intrinsic Anomalous Hall Effect

Changgan Zeng,¹ Yugui Yao,² Qian Niu,³ and Hanno H. Weitering^{1,4}

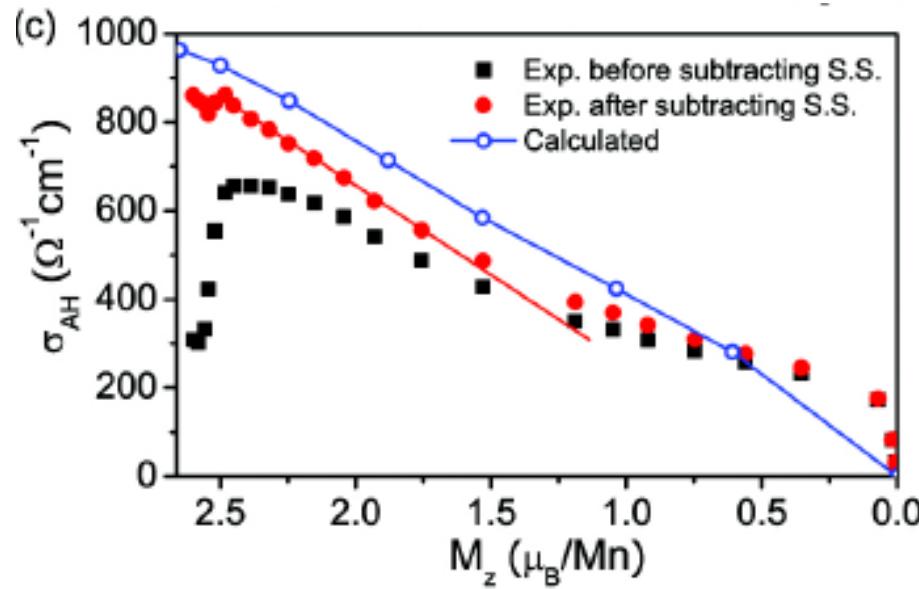
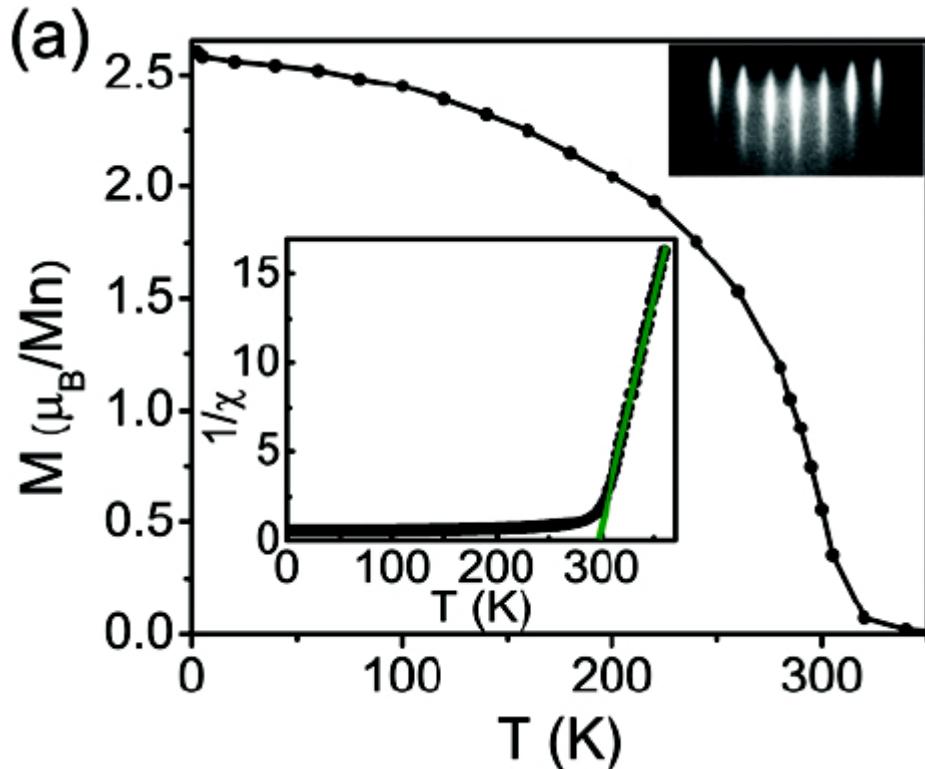
¹*Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996, USA*

²*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China*

³*Department of Physics, University of Texas, Austin, Texas 78712, USA*

⁴*Condensed Matter Sciences Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*

(Received 21 October 2005; published 25 January 2006)

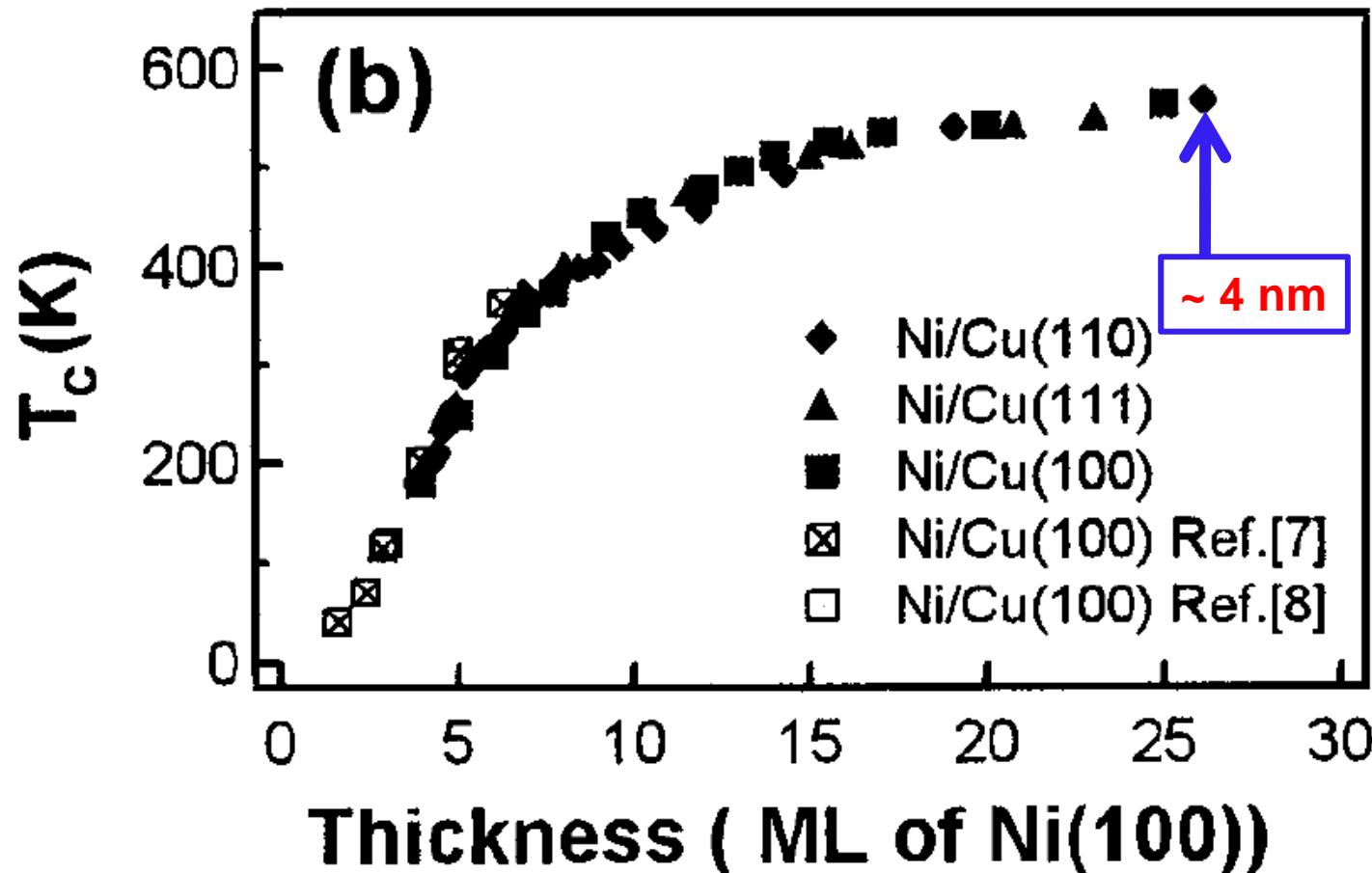


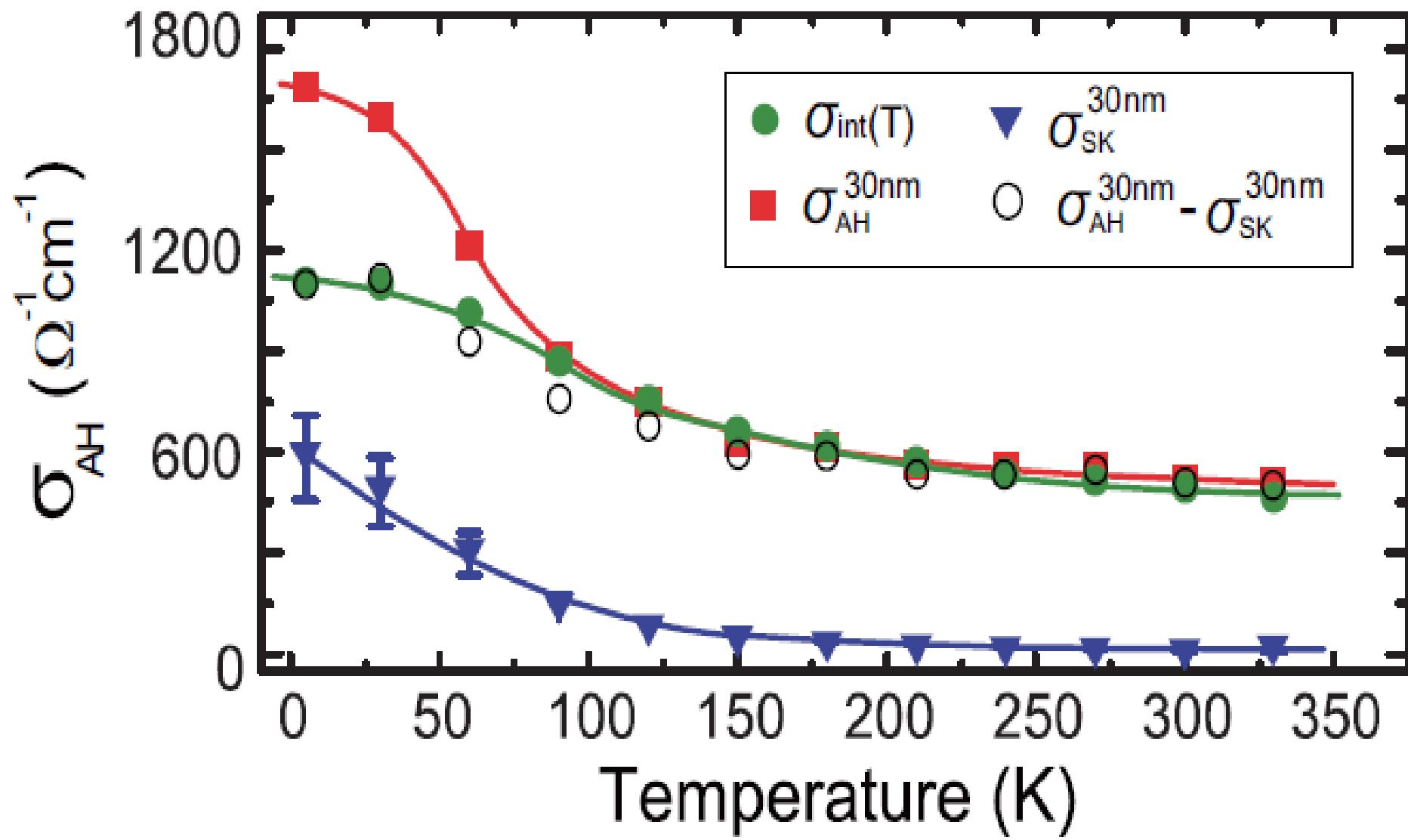
Thickness-Dependent Curie Temperatures of Ultrathin Magnetic Films: Effect of the Range of Spin-Spin Interactions

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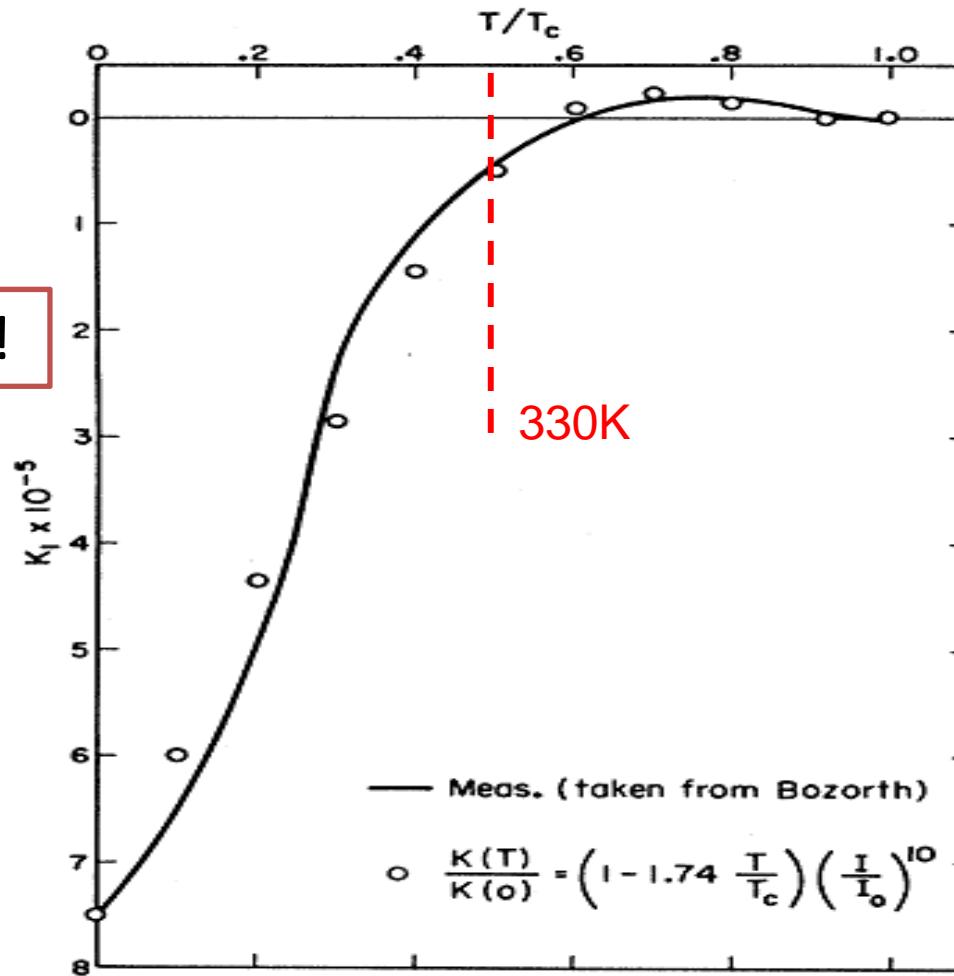
(Received 31 October 2000)





Temperature dependent magneto-crystalline anisotropy

93% decrease!



W. J. Carr, Phy. Rev. 109 (1958) 1971

*MAGNETIC ANISOTROPY OF NICKEL*E. I. KONDORSKI¹ and E. STRAUBE

Moscow State University

Submitted February 9, 1972

Zh. Eksp. Teor. Fiz. 63, 356-365 (July, 1972)

A theory of magnetic anisotropy of nickel is developed in which its band structure and shape of the Fermi surface are taken into account. It is shown that the theoretical values of the magnetic anisotropy constant at absolute zero temperature are in satisfactory agreement with the experimental value for the helium temperature range. It is shown that the main contribution to the anisotropic part of the energy (AE) is due to the Brillouin band regions which would contain degenerate or quasi-degenerate states in the absence of spin-orbit interaction. It is necessary in this case that the Fermi levels be located within one or several, but not all, bands with degenerate or quasidegenerate states. The contributions to the AE from other regions of the Brillouin band are smaller by several orders of magnitude than the main contributions mentioned above.

Band structure of nickel: Spin-orbit coupling, the Fermi surface, and the optical conductivity*

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(Received 7 December 1973)

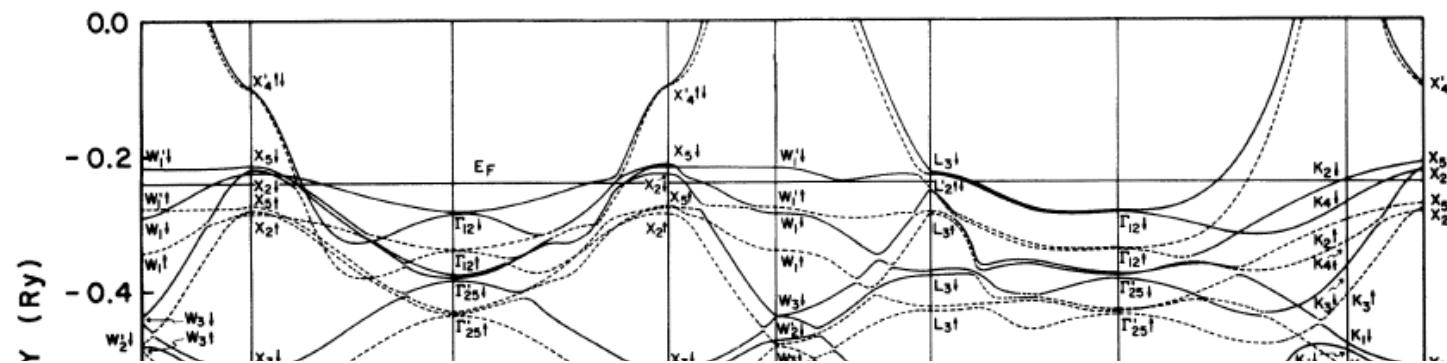


TABLE I. Energy levels at symmetry points (Ry).

Band	$\Gamma(0, 0, 0)$	$X(1, 0, 0)$	$X(0, 0, 1)$	$L(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$W(1, \frac{1}{2}, 0)$	$W(1, 0, \frac{1}{2})$	$K(\frac{3}{4}, \frac{3}{4}, 0)$	$K(\frac{3}{4}, 0, \frac{3}{4})$
12	-0.2850 (Γ_{12}^\dagger)	-0.0983 (X_4^\dagger)	-0.0983 (X_4^\dagger)	-0.2264 (L_3^\dagger)	0.2978 (W_3^\dagger)	0.3057 (W_3^\dagger)	0.1709 (K_3^\dagger)	0.1709 (K_3^\dagger)
11	-0.2851 (Γ_{12}^\dagger)	-0.0993 (X_4^\dagger)	-0.0993 (X_4^\dagger)	-0.2300 (L_3^\dagger)	0.2978 (W_3^\dagger)	0.2937 (W_3^\dagger)	0.1519 (K_3^\dagger)	0.1519 (K_3^\dagger)
10	-0.3408 (Γ_{12}^\dagger)	-0.2149 (X_5^\dagger)	-0.2124 (X_5^\dagger)	-0.2520 (L_2^\dagger)	-0.2164 (W_1^\dagger)	-0.2164 (W_1^\dagger)	-0.2368 (K_2^\dagger)	-0.2367 (K_2^\dagger)
9	-0.3413 (Γ_{12}^\dagger)	-0.2160 (X_5^\dagger)	-0.2203 (X_5^\dagger)	-0.2521 (L_2^\dagger)	-0.2766 (W_1^\dagger)	-0.2766 (W_1^\dagger)	-0.2703 (K_4^\dagger)	-0.2707 (K_4^\dagger)
8	-0.3778 (Γ_{25}^\dagger)	-0.2272 (X_2^\dagger)	-0.2255 (X_2^\dagger)	-0.2860 (L_3^\dagger)	-0.2881 (W_1^\dagger)	-0.2881 (W_1^\dagger)	-0.2967 (K_2^\dagger)	-0.2962 (K_2^\dagger)
7	-0.3808 (Γ_{25}^\dagger)	-0.2758 (X_5^\dagger)	-0.2736 (X_5^\dagger)	-0.2898 (L_3^\dagger)	-0.3429 (W_1^\dagger)	-0.3428 (W_1^\dagger)	-0.3272 (K_4^\dagger)	-0.3273 (K_4^\dagger)
6	-0.3854 (Γ_{25}^\dagger)	-0.2773 (X_5^\dagger)	-0.2805 (X_5^\dagger)	-0.3736 (L_3^\dagger)	-0.4387 (W_3^\dagger)	-0.4357 (W_3^\dagger)	-0.3670 (K_3^\dagger)	-0.3669 (K_3^\dagger)
5	-0.4304 (Γ_{25}^\dagger)	-0.2869 (X_2^\dagger)	-0.2859 (X_2^\dagger)	-0.3803 (L_3^\dagger)	-0.4389 (W_3^\dagger)	-0.4428 (W_3^\dagger)	-0.4090 (K_3^\dagger)	-0.4091 (K_3^\dagger)
4	-0.4338 (Γ_{25}^\dagger)	-0.5179 (X_3^\dagger)	-0.5178 (X_3^\dagger)	-0.4263 (L_3^\dagger)	-0.4766 (W_2^\dagger)	-0.4736 (W_2^\dagger)	-0.4837 (K_1^\dagger)	-0.4836 (K_1^\dagger)
3	-0.4365 (Γ_{25}^\dagger)	-0.5398 (X_1^\dagger)	-0.5398 (X_1^\dagger)	-0.4330 (L_3^\dagger)	-0.4815 (W_3^\dagger)	-0.4823 (W_3^\dagger)	-0.4929 (K_1^\dagger)	-0.4929 (K_1^\dagger)
2	-0.9144 (Γ_1^\dagger)	-0.5619 (X_3^\dagger)	-0.5619 (X_3^\dagger)	-0.5873 (L_1^\dagger)	-0.4833 (W_3^\dagger)	-0.4848 (W_3^\dagger)	-0.5219 (K_1^\dagger)	-0.5219 (K_1^\dagger)
1	-0.9155 (Γ_1^\dagger)	-0.5775 (X_1^\dagger)	-0.5774 (X_1^\dagger)	-0.6178 (L_1^\dagger)	-0.5183 (W_2^\dagger)	-0.5182 (W_2^\dagger)	-0.5359 (K_1^\dagger)	-0.5359 (K_1^\dagger)

Intrinsic Spin Hall Effect in Platinum: First-Principles Calculations

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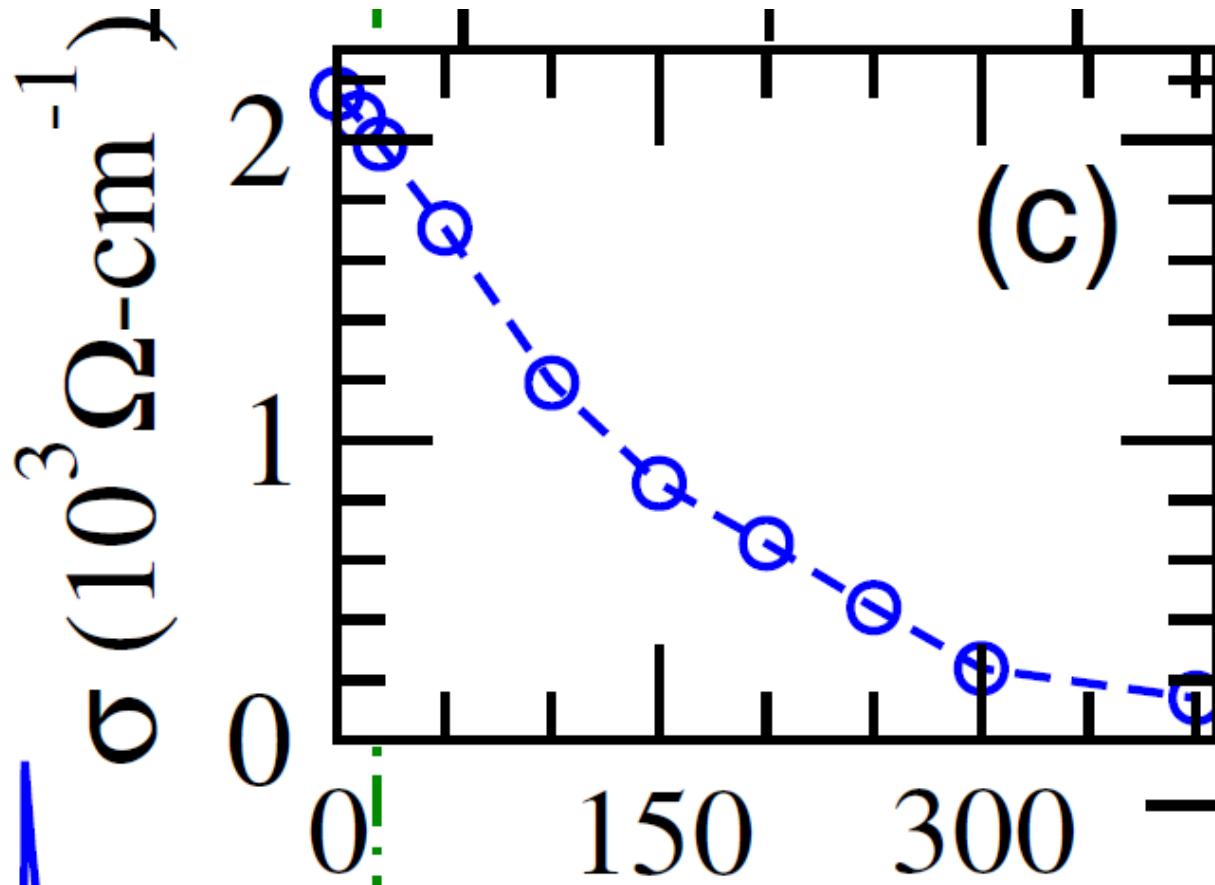
²*Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan*

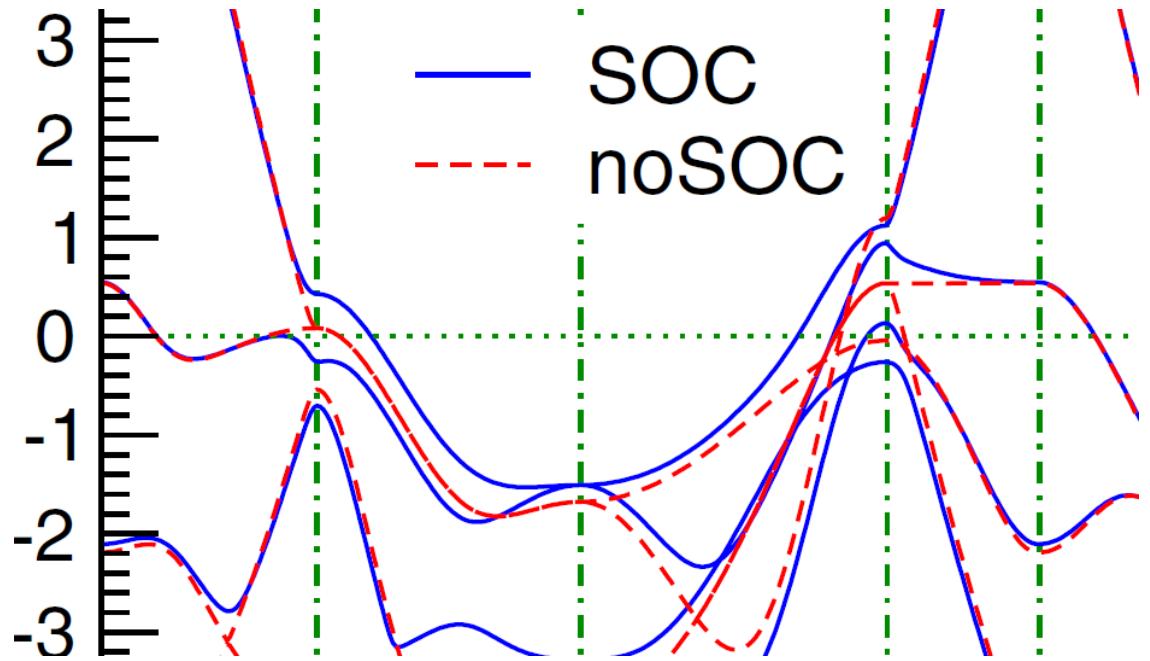
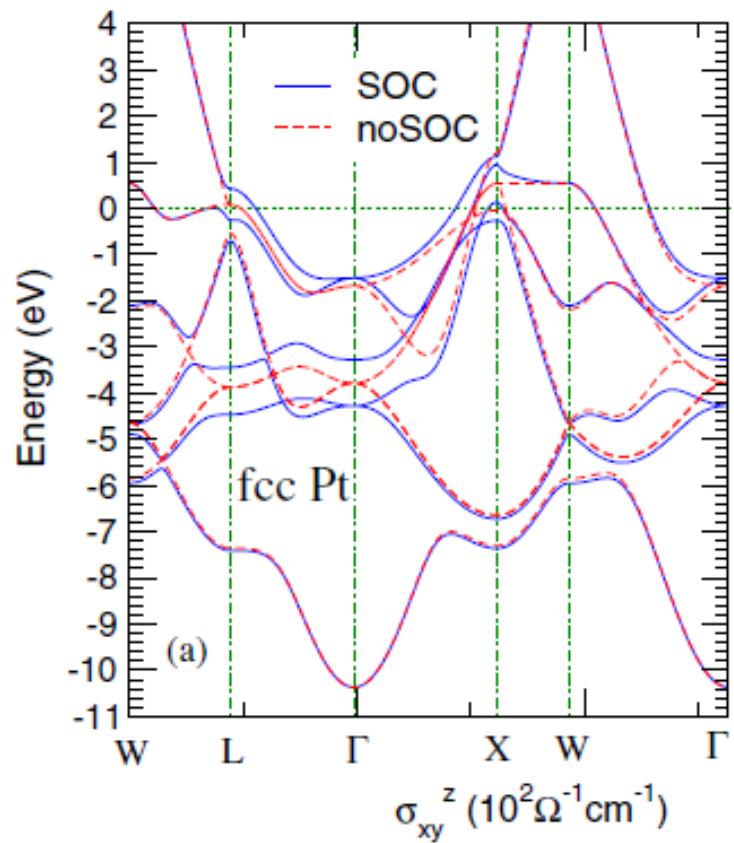
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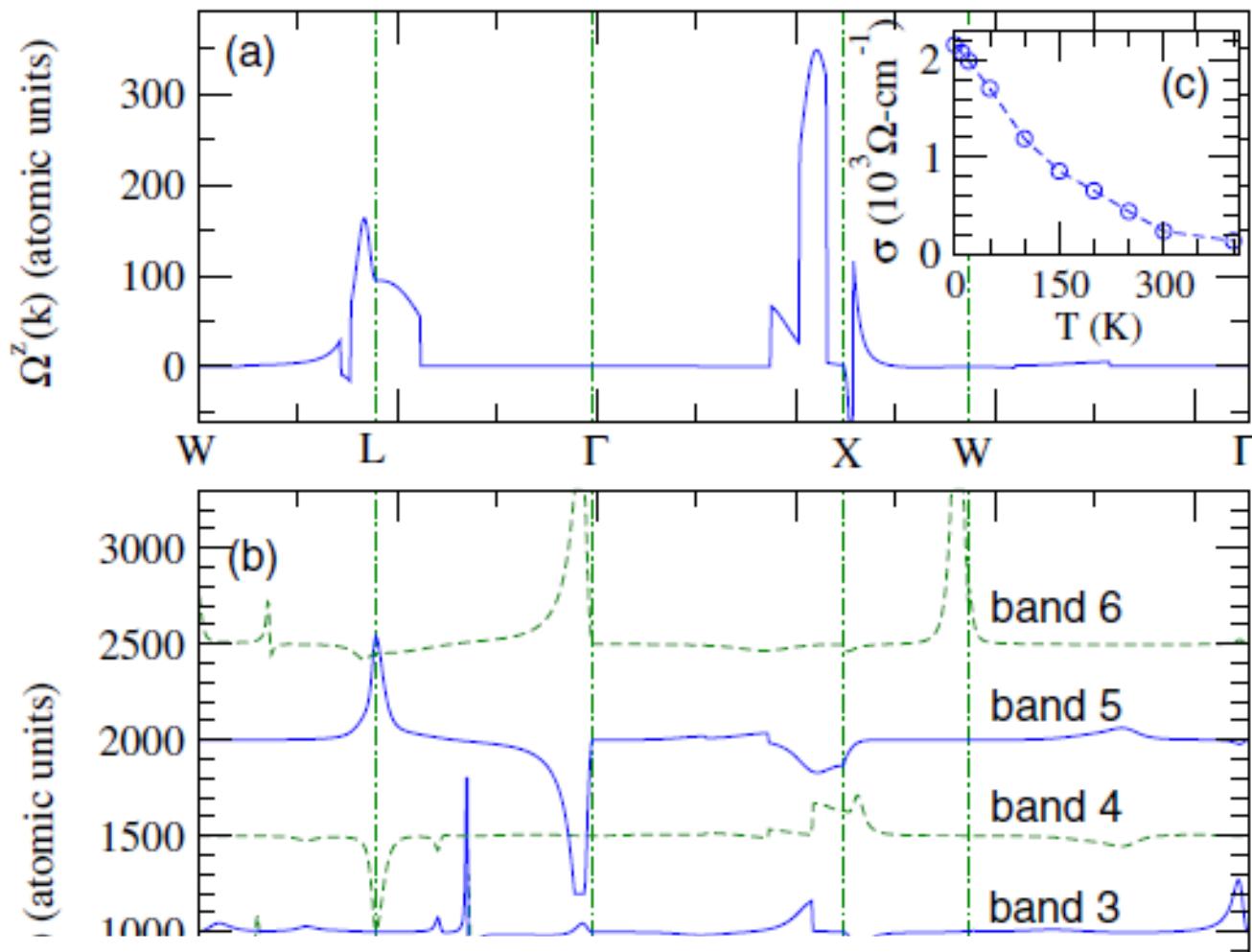
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This rather strong temperature dependence is also due to the near degeneracies since the small energy scale is relevant to the SHC there.

3. Conclusions

Intrinsic and Extrinsic in the AHE

$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b(T) \rho_{xx}^2$$

Extrinsic

Intrinsic

Y. Tian, L. Ye, X.F. Jin, Phys. Rev. Lett., 103, 087206 (2009)

L. Ye, Y. Tian, X.F. Jin, D. Xiao, arXiv: 1105.5664



THANK YOU