Angular Momenta, Geometric Phases, and Spin-Orbit Interactions of Light and Electron Waves

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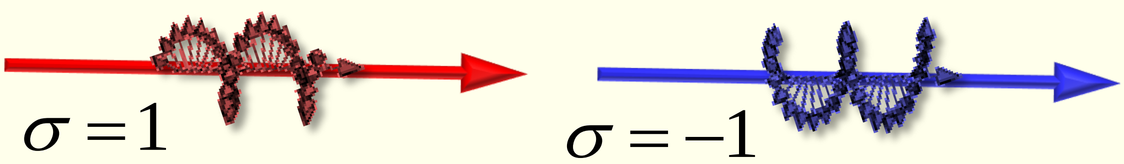
- **Basic concepts: Angular momenta, Berry phase, Spin-orbit interactions, Hall effects**
- Theory of photon AM
- Application to Bessel beams
- Electron vortex beams
- Theory of electron AM

Angular momentum of light

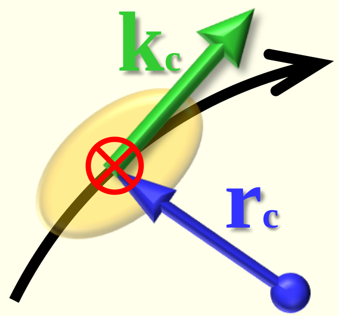
1. Intrinsic spin AM (polarization)



$$\mathbf{S} = \sigma\,\boldsymbol{\kappa}_c$$

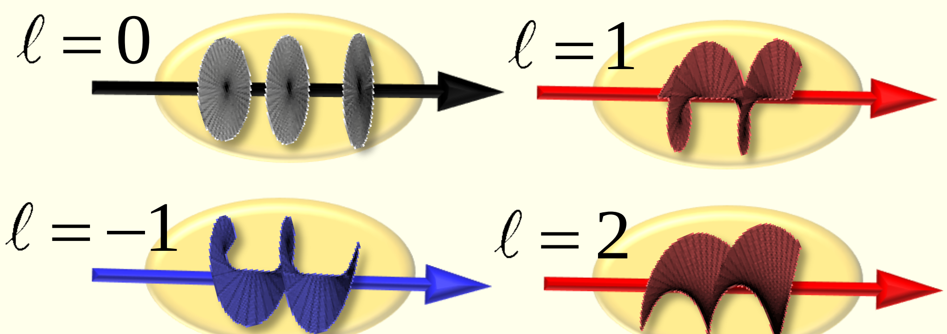
$$\boldsymbol{\kappa}_c = \mathbf{k}_c / k_c$$

2. Extrinsic orbital AM (trajectory)



$$\mathbf{L}_{ext} = \mathbf{r}_c \times \mathbf{k}_c$$

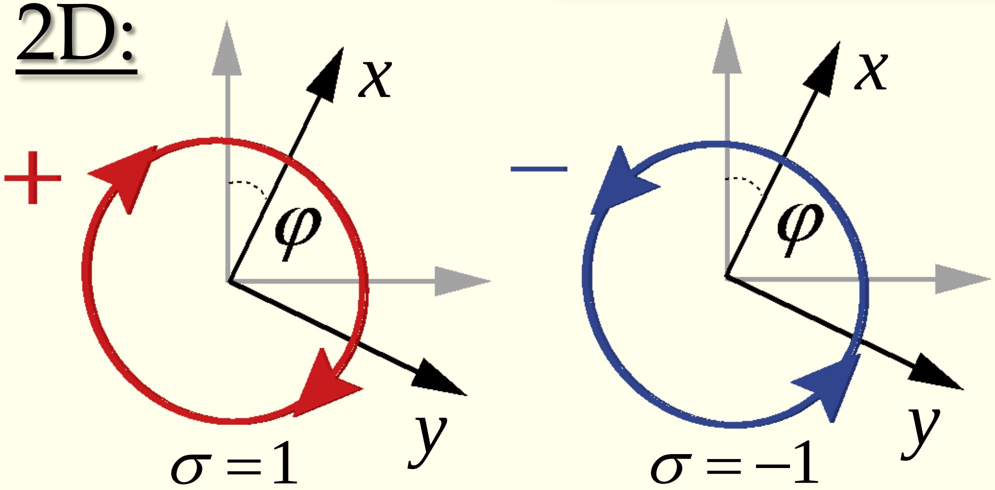
3. Intrinsic orbital AM (vortex)



$$\mathbf{L}_{int} \propto \int (\mathbf{r} - \mathbf{r}_c) \times \mathbf{k}\, dV$$
$$\propto \ell\, \boldsymbol{\kappa}_c$$

Geometric phase

2D:

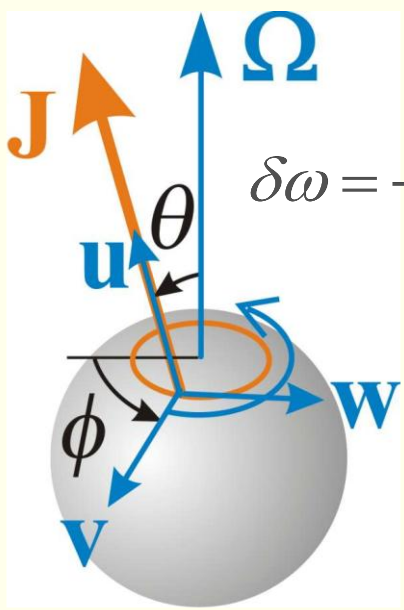


$$\mathbf{e}^{\sigma} = \left(\mathbf{e}_x + i\sigma\mathbf{e}_y\right)$$

$$\mathbf{e}^{\sigma} \to e^{i\Phi}\mathbf{e}^{\sigma},\; E^{\sigma} \to e^{-i\Phi}E^{\sigma}$$

$$\Phi = -\sigma\varphi$$

3D:

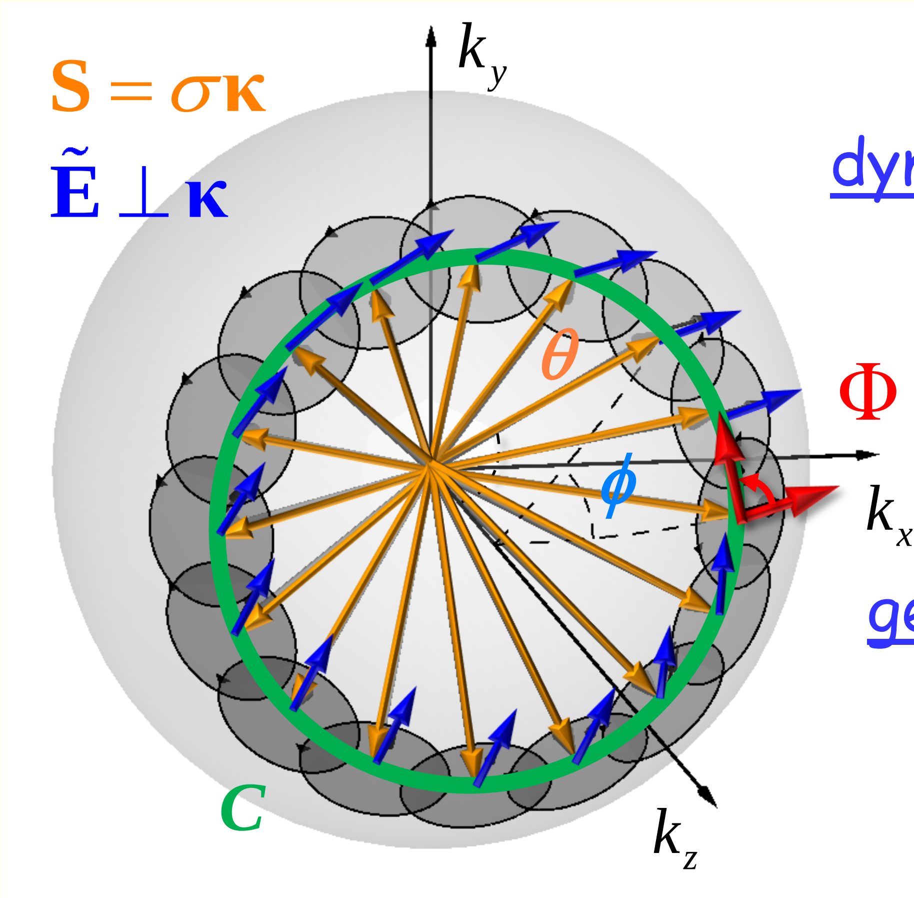


$$\delta\omega = -\mathbf{J}\cdot\mathbf{\Omega}$$

$$\Phi = -\int \mathbf{J}\cdot\mathbf{\Omega}_{\zeta}\, d\zeta$$

AM-rotation coupling:
Coriolis / angular-Doppler effect

Geometric phase



$$\left[\mathbf{e}^{+}, \mathbf{e}^{-}, \boldsymbol{\kappa}\right]: \mathbf{e}^{\sigma}(\boldsymbol{\kappa}) = \left(\mathbf{e}_{\theta} + i\sigma \mathbf{e}_{\phi}\right) e^{i\sigma\phi}$$

dynamics, **S**:

$$\Phi = 2\pi\sigma - \int d\phi\, S_z$$
$$= 2\pi\sigma\left(1 - \cos\theta\right)$$

geometry, **E**:

$$\Phi = \sigma \int_C \mathcal{A}(\mathbf{k}) \cdot d\mathbf{k}$$

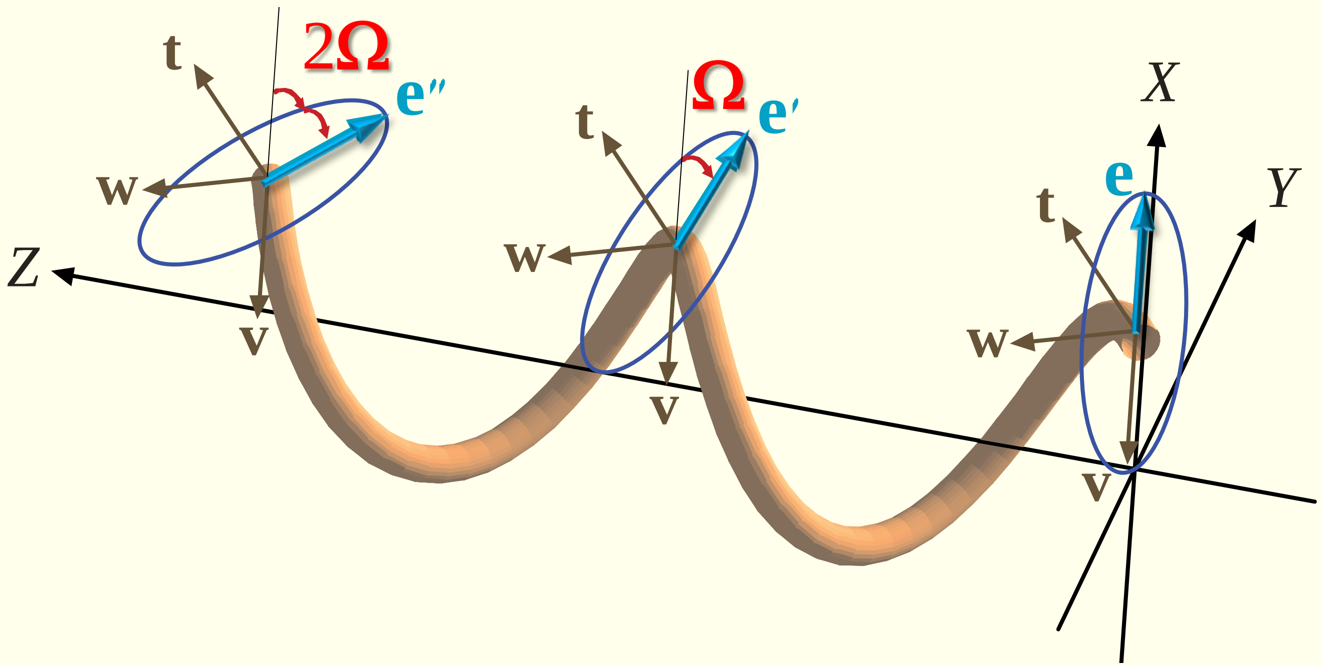
$$\mathcal{A} = \frac{1 - \cos\theta}{k \sin\theta} \mathbf{e}_{\phi}, \quad \mathcal{F} = \partial_{\mathbf{k}} \times \mathcal{A} = \frac{\mathbf{k}}{k^3}$$

– Berry connection, curvature (parallel transport)

Geometric phase

$$\Phi = \sigma \oint_C \mathcal{A} \cdot d\mathbf{k} = \sigma \int_S \mathcal{F}\, d\mathbf{s} = -\sigma\Omega$$

– Berry phase for light



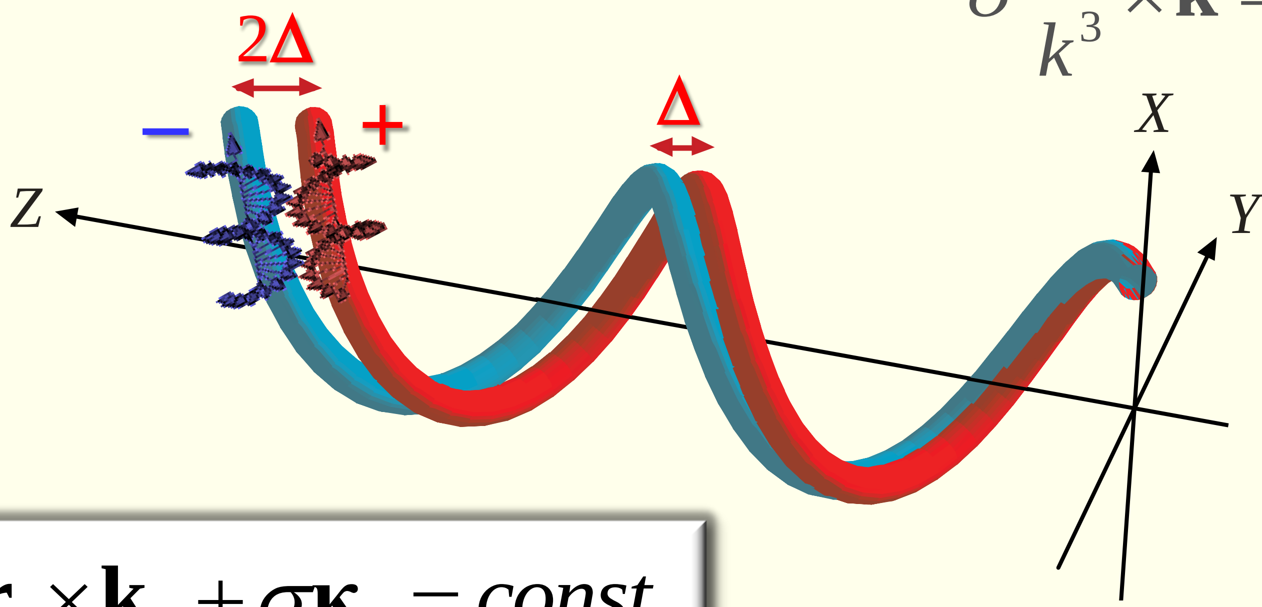
Rytov, 1938; Vladimirskiy, 1941; Ross, 1984; Tomita, Chiao, Wu, PRL 1986

Spin-Hall effect of light

$$\dot{\mathbf{k}}_c = k\nabla \ln n\,, \quad \dot{\mathbf{r}}_c = \mathbf{t} + \sigma\mathcal{F}\times\dot{\mathbf{k}}$$

– ray equations (equations of motion)

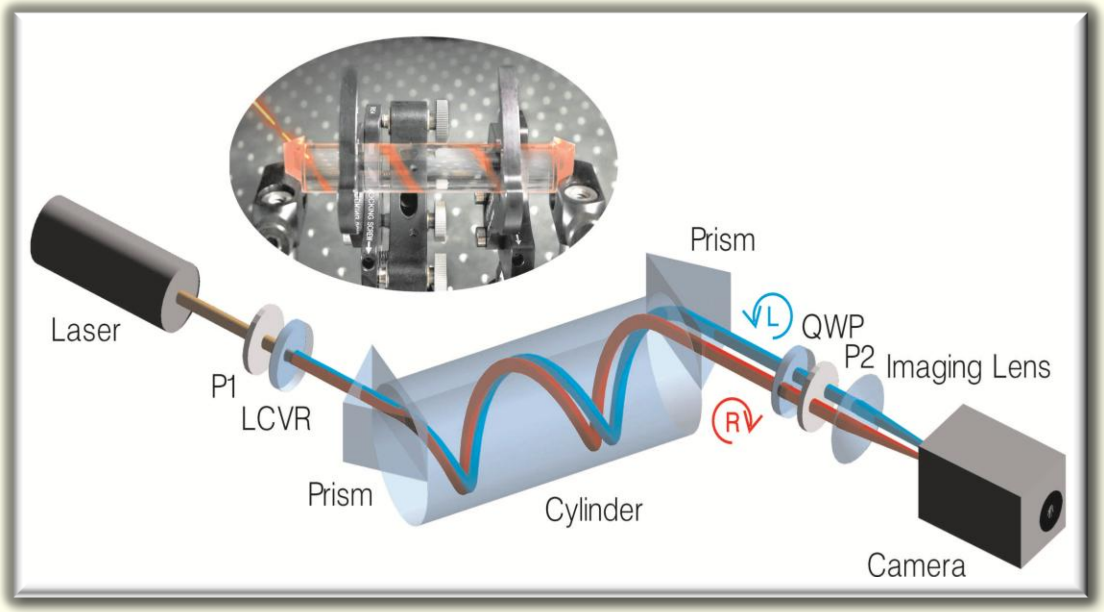
$$\sigma\frac{\mathbf{k}}{k^3}\times\dot{\mathbf{k}} = \lambda\!\!\!^{-}\mathbf{S}\times\dot{\boldsymbol{\kappa}}$$



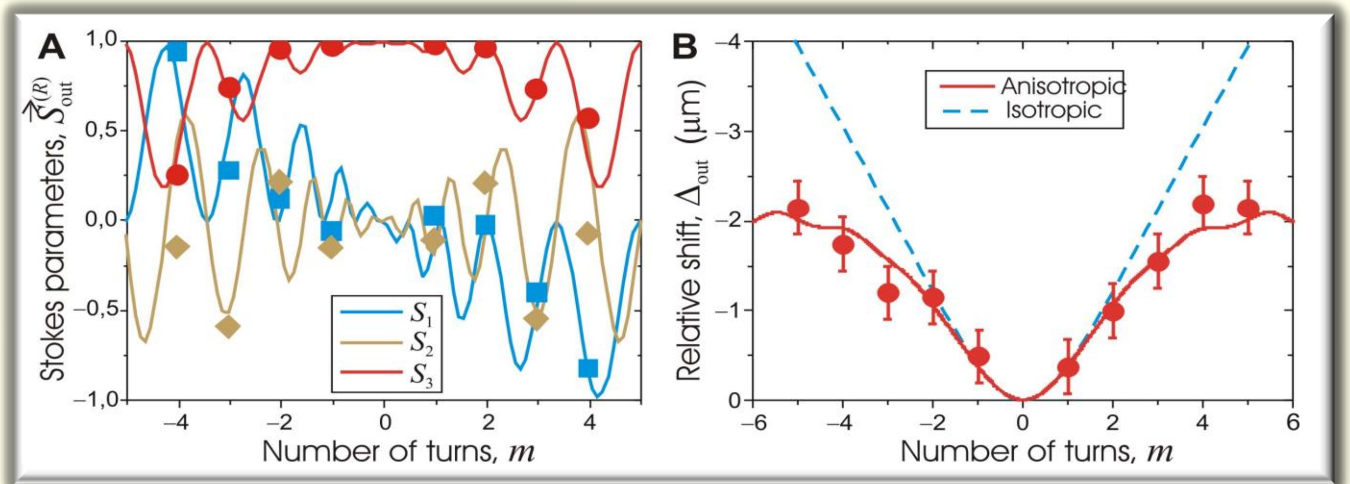
$$\mathbf{J} = \mathbf{r}_c\times\mathbf{k}_c + \sigma\boldsymbol{\kappa}_c = const$$

Liberman & Zeldovich, PRA 1992;
Bliokh & Bliokh, PLA 2004, PRE 2004; Onoda, Murakami, Nagaosa, PRL 2004

Spin-Hall effect of light

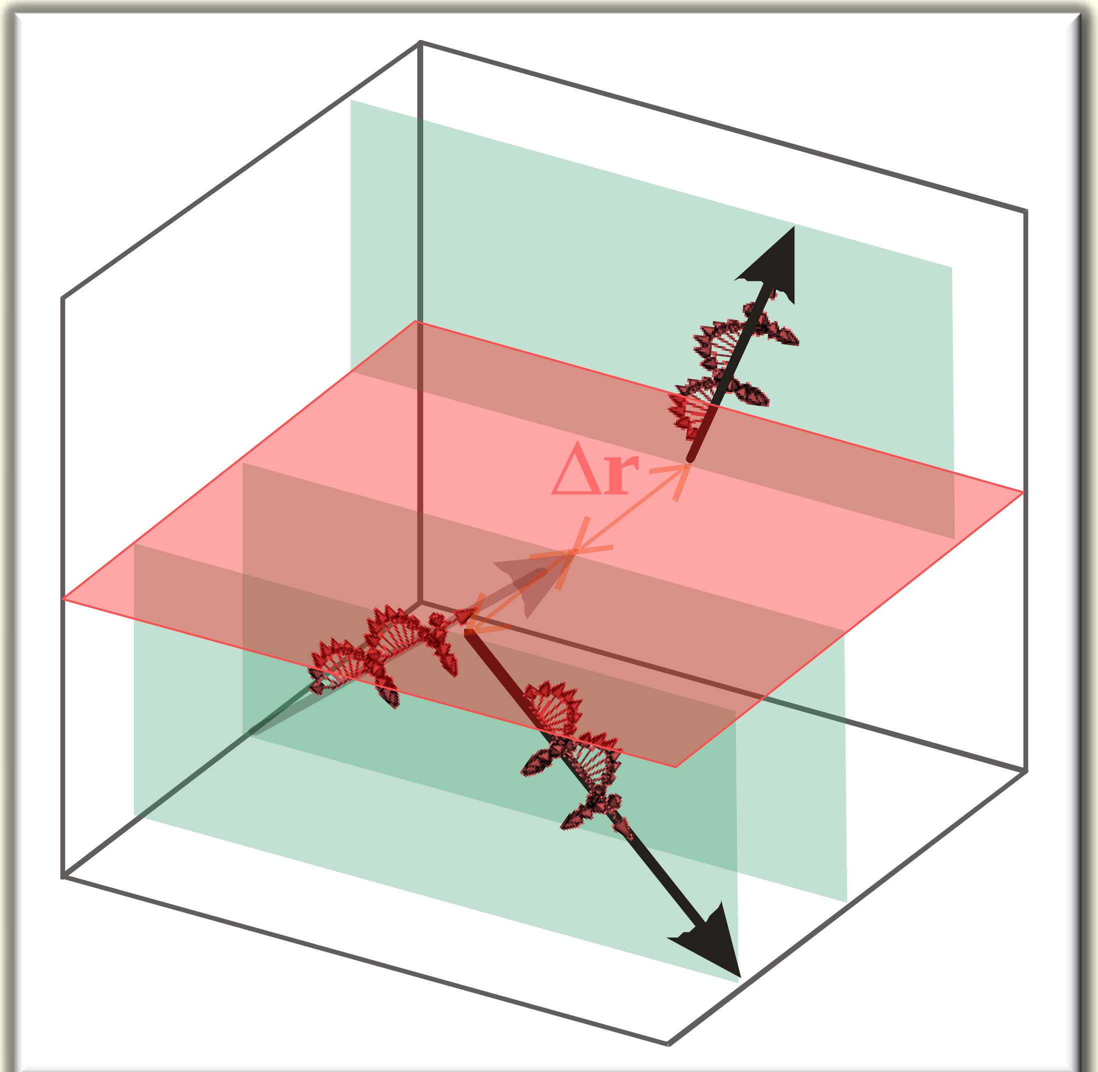


- Berry phase
- Spin-Hall effect
- Anisotropy



Bliokh et al., Nature Photon. 2008

Spin-Hall effect of light

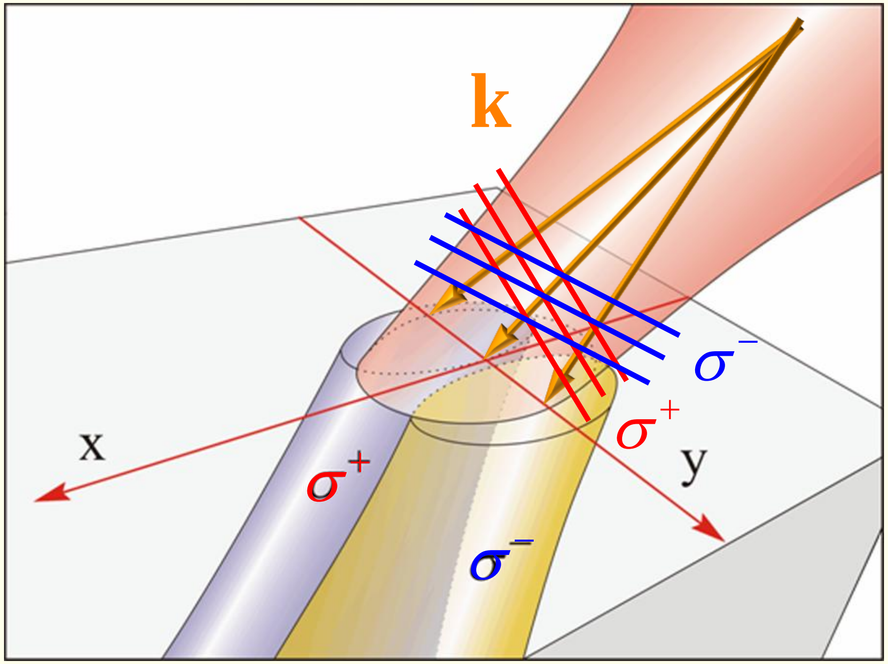


$$\Delta \mathbf{r} \propto \sigma \lambda$$

Imbert–Fedorov transverse shift

Fedorov, 1955; Imbert, 1972;
……
Onoda et al., PRL 2004;
Bliokh & Bliokh, PRL 2006;
Hosten & Kwiat, Science 2008

Spin-Hall effect of light



geometry, phase:

$$\mathbf{k}_{\rm c} \to \mathbf{k}_{\rm c} + k_y \mathbf{e}_y :$$

$$\hat{R}_z\left(\kappa_y / \sin\theta\right), \quad S_z = \sigma\cos\theta$$

$\Rightarrow$

$$\Phi = -\sigma\kappa_y \cot\theta$$

$\Rightarrow$

$$Y = -\partial_{k_y}\Phi = k^{-1}\sigma\cot\theta$$

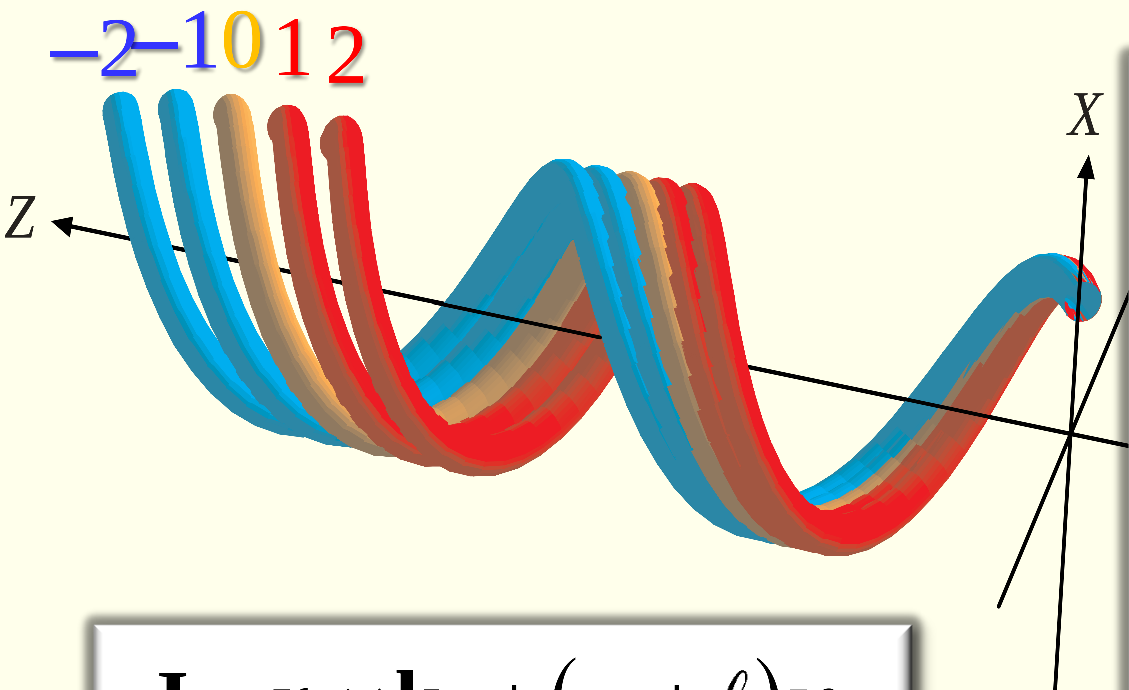
dynamics, AM:

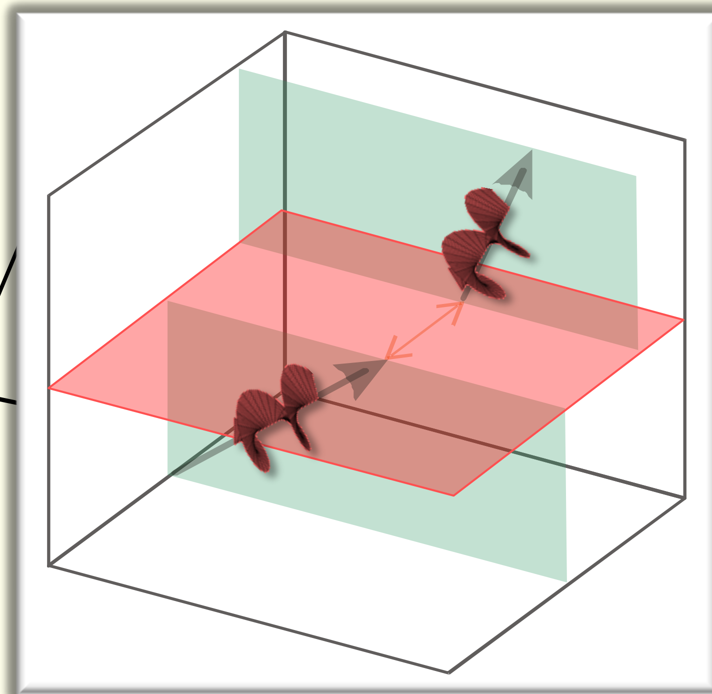
$$\mathbf{J} = \mathbf{L}_{\rm ext} + \mathbf{S} : \quad \delta J_z = -kY\sin\theta + \sigma\cos\theta \;\Rightarrow\; \delta J_z = 0$$

Orbital-Hall effect of light

$$\sigma \to \sigma + \ell$$

– vortex-depndent shifts and *orbit-orbit* interaction





$$\mathbf{J} = \mathbf{r}_c \times \mathbf{k}_c + (\sigma + \ell)\boldsymbol{\kappa}_c$$

Bliokh, PRL 2006

Fedoseyev, Opt. Commun. 2001; Dasgupta & Gupta, Opt. Commun. 2006

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Angular momentum of light

$$\hat{\mathbf{J}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} + \hat{\mathbf{S}} \equiv \hat{\mathbf{L}} + \hat{\mathbf{S}}$$

– total AM = OAM +SAM?

$$\hat{\mathbf{r}} = i\partial_{\mathbf{k}}, \quad \hat{\mathbf{p}} = \mathbf{k}, \quad \left(\hat{S}_a\right)_{ij} = -i\varepsilon_{aij}$$

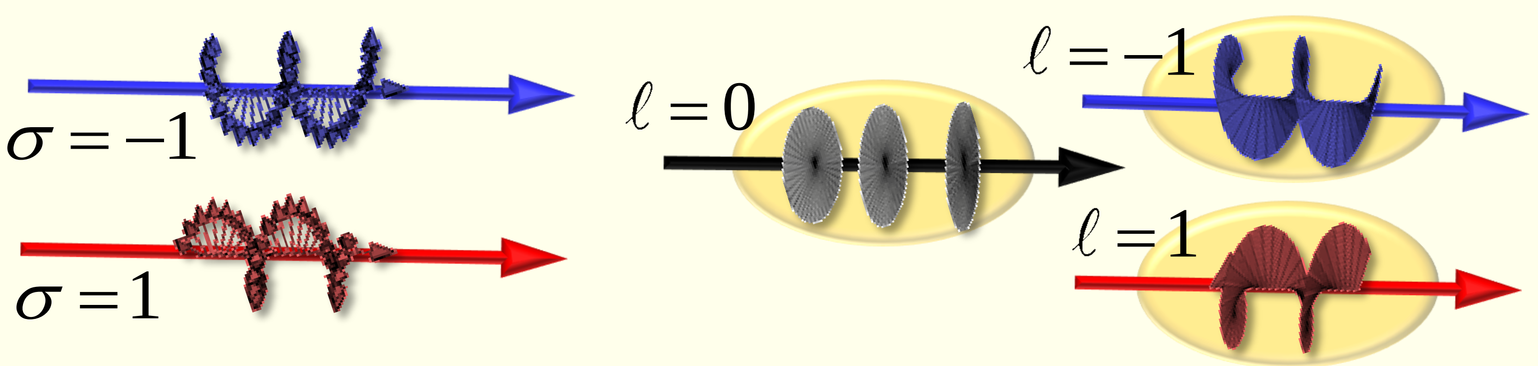
$$\hat{L}_z = -i\partial_\phi, \quad \left(\hat{S}_z\right)_{ij} = -i\varepsilon_{zij}$$

$$\mathbf{E}_{\ell\sigma} \propto \left(\mathbf{e}_x + i\sigma\mathbf{e}_y\right)e^{i\ell\phi}$$

– z-components and paraxial eigenmodes:

σ - polarization,

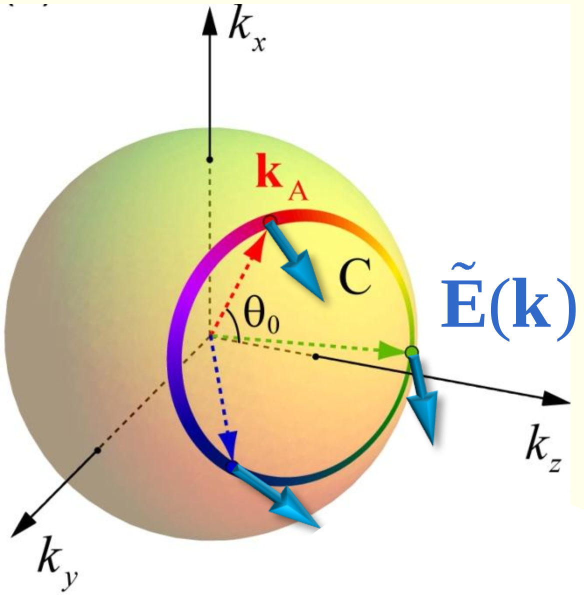
ℓ - vortex



Nonparaxial problem 1

$$\tilde{\mathbf{E}} \cdot \mathbf{k} = 0 \Rightarrow \tilde{\mathbf{E}} \perp \boldsymbol{\kappa}$$

– transversality constraint:
3D → 2D



"the separation of the total AM into orbital and spin parts has restricted physical meaning. ... States with definite values of OAM and SAM do not satisfy the condition of transversality in the general case."

A. I. Akhiezer, V. B. Berestetskii, "Quantum Electrodynamics" (1965)

$$\hat{\mathbf{L}}\tilde{\mathbf{E}} \not\perp \boldsymbol{\kappa}, \quad \hat{\mathbf{S}}\tilde{\mathbf{E}} \not\perp \boldsymbol{\kappa}$$

though $\hat{\mathbf{J}}\tilde{\mathbf{E}} \perp \boldsymbol{\kappa}$

Nonparaxial problem 2

$$\boxed{\tilde{\mathbf{E}}_{\ell\sigma} \propto \left(\mathbf{e}_\theta + i\sigma \mathbf{e}_\phi\right) e^{i\sigma\phi} e^{i\ell\phi}} \equiv \mathbf{e}^{\sigma}\left(\boldsymbol{\kappa}\right) e^{i\ell\phi}$$

– nonparaxial vortex

$\Rightarrow$

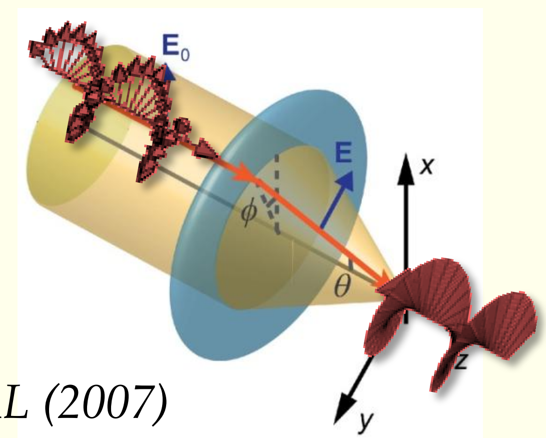
$$\boxed{L_z = \ell + \gamma\sigma, \;\; S_z = \left(1 - \gamma\right)\sigma}$$

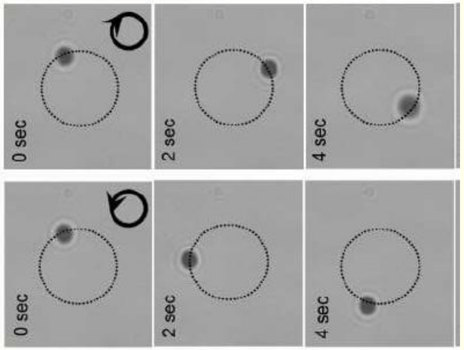
though $J_z = \ell + \sigma$

"in the general nonparaxial case there is no separation into ℓ-dependent orbital and σ-dependent spin part of AM"

S. M. Barnett, L. Allen, Opt. Commun. (1994)

spin-to-orbital AM conversion?..





Y. Zhao et al., PRL (2007)

Modified AM and position operators

$\hat{\mathbf{J}} = \hat{\mathbf{L}}' + \hat{\mathbf{S}}'$, compatible with transversality:

$$\hat{\mathbf{S}}' = \hat{\mathbf{S}} - \hat{\mathbf{\Delta}} = \boldsymbol{\kappa}\left(\boldsymbol{\kappa}\cdot\hat{\mathbf{S}}\right) \equiv \boldsymbol{\kappa}\hat{\sigma}$$

$$\hat{\mathbf{L}}' = \hat{\mathbf{L}} + \hat{\mathbf{\Delta}} = \hat{\mathbf{r}}' \times \mathbf{k}$$

$$\hat{\mathbf{\Delta}} = -\boldsymbol{\kappa}\times\left(\boldsymbol{\kappa}\times\hat{\mathbf{S}}\right)$$

– projected SAM and OAM

cf. S.J. van Enk, G. Nienhuis, JMO (1994)

$\Rightarrow$

$$\hat{\mathbf{r}}' = \hat{\mathbf{r}} + \left(\mathbf{k}\times\hat{\mathbf{S}}\right)/k^2$$

– spin-dependent position

M.H.L. Pryce, PRSLA (1948), etc.

Spin-dependent OAM and position signify the **spin-orbit interaction (SOI) of light**: (i) spin-to-orbital AM conversion, (ii) spin Hall effect.

Helicity representation

$$\hat{U}(\boldsymbol{\kappa}):\ (\mathbf{e}_x,\mathbf{e}_y,\mathbf{e}_z)\rightarrow(\mathbf{e}^+,\mathbf{e}^-,\boldsymbol{\kappa})\ \Rightarrow\ \hat{\mathbf{O}}\rightarrow\hat{U}^\dagger\hat{\mathbf{O}}\hat{U}$$

$$\hat{\mathbf{S}}'=\boldsymbol{\kappa}\hat{\sigma}$$

$$\hat{\mathbf{L}}'=\hat{\mathbf{L}}-\hat{\sigma}\boldsymbol{\mathcal{A}}\times\mathbf{k}$$

$$\hat{\mathbf{r}}'=\hat{\mathbf{r}}-\hat{\sigma}\boldsymbol{\mathcal{A}}$$

– all operators become diagonal !

$$\hat{\sigma}=\mathrm{diag}(1,-1,0)$$

$\boldsymbol{\mathcal{A}}(\mathbf{k})$ – Berry connection

cf. I. Bialynicki-Birula, Z. Bialynicka-Birula (1987), B.-S. Skagerstam (1992), A. Berard, H. Mohrbach (2006)

3D → 2D reduction and diagonalization:

$$\begin{pmatrix} \perp & \perp & 0 \\ \perp & \perp & 0 \\ 0 & 0 & \parallel \end{pmatrix}$$

$$A^\sigma=\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\rightarrow\sigma=1,\,-1$$

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Bessel beams in free space

$\tilde{E}^{\sigma}_{\ell} = A^{\sigma}\delta(\theta-\theta_0)e^{i\ell\phi}$ – field $\mathbf{O} = \left\langle \tilde{E}^{\sigma} \middle| \hat{\mathbf{O}}' \middle| \tilde{E}^{\sigma} \right\rangle$ – mean values

$$S_z = \sigma(1-\Phi)\,, \quad L_z = \ell + \sigma\Phi$$

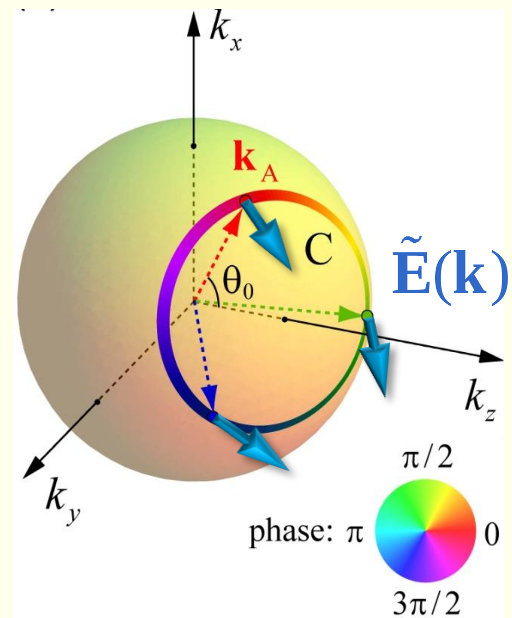
– SAM and OAM for Bessel beam

$$\Phi = \oint_C \mathcal{A}\cdot d\mathbf{k} = 2\pi(1-\cos\theta_0) \sim \theta_0^2$$

– Berry phase

$\Rightarrow$

The spin-to-orbit AM conversion in nonparaxial fields originates from the Berry phase associated with the azimuthal distribution of partial waves.



Quantization of caustics

$$I_\ell^\sigma(r) \propto \left[a^2 J_\ell^2(\tilde{r}) + b^2 J_{\ell+2\sigma}^2(\tilde{r}) + 2ab J_{\ell+\sigma}^2(\tilde{r}) \right]$$

– spin-dependent intensity $(b = \Phi/2,\ a = 1 - \Phi/2)$

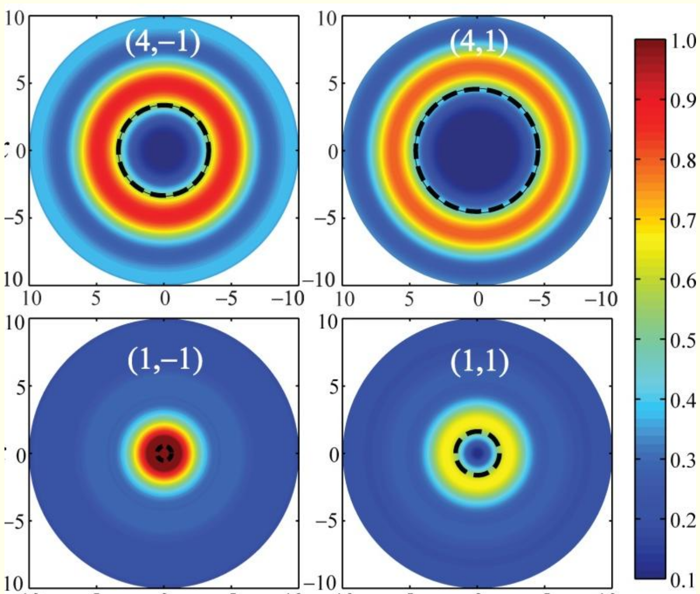
$$k_\perp R_\ell^\sigma = \left| \ell + \sigma\Phi \right|$$

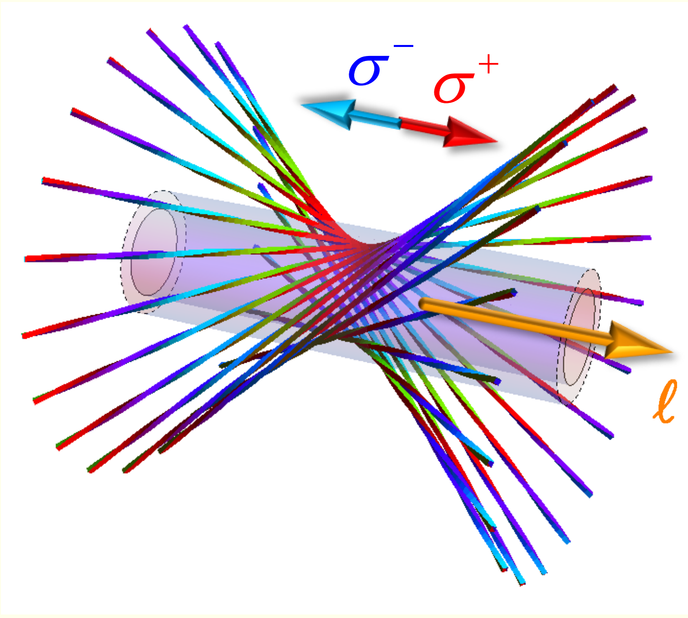
– quantized GO caustic:
fine SOI splitting !

$\sigma = -1$ $\sigma = 1$

$\ell = 4$

$\ell = 1$

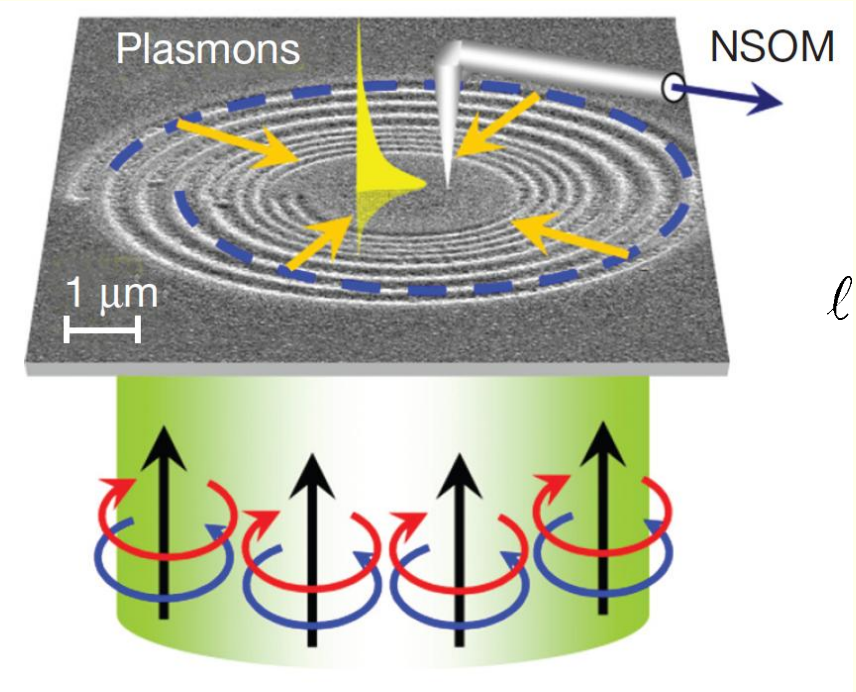




Plasmonic experiment

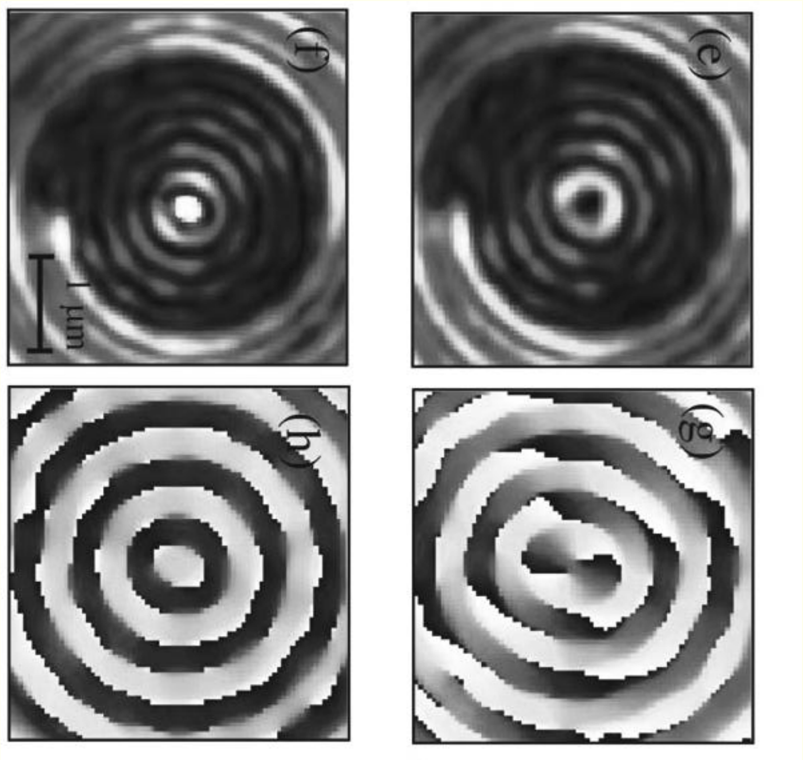
$\underline{\theta_0 = \pi/2}$, $\underline{\Phi_B = 2\pi}$

$$\ell + \sigma\Phi_B = \ell + \sigma$$



$\sigma = -1$ $\sigma = 1$

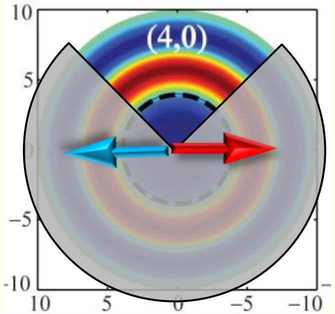
$\ell = 1$



– circular plasmonic lens generates Bessel modes

Y. Gorodetski et al., PRL (2008)

Spin and orbital Hall effects



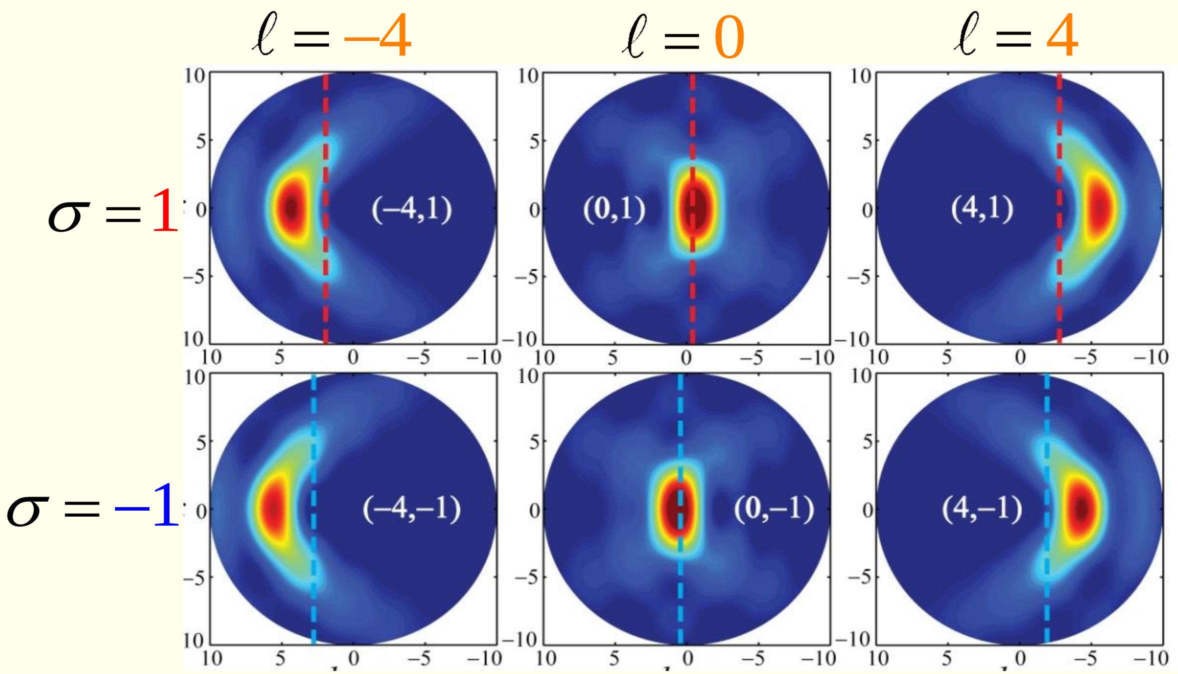
– azimuthally truncated field (symmetry breaking along x)

$\phi \in (-\delta, \delta)$

B. Zel'dovich et al. (1994), K.Y. Bliokh et al. (2008)

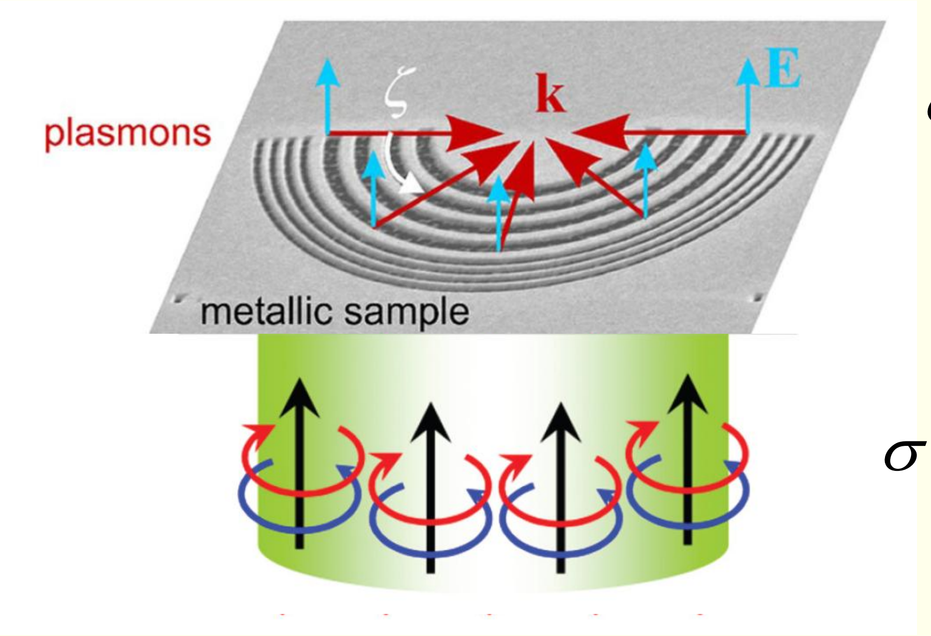
$$\Rightarrow \quad k_\perp Y_\ell^\sigma = -\gamma\left(\ell + \sigma \Phi_B\right)$$

– orbital and spin Hall effects of light



Plasmonic experiment

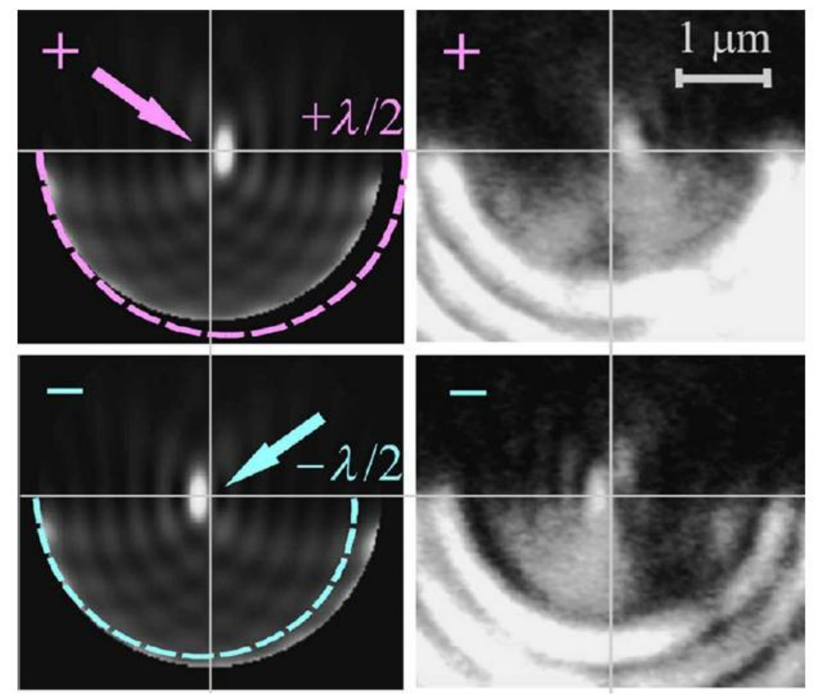
Plasmonic half-lens produces spin Hall effect:



$\ell = 0$

$\sigma = 1$

$\sigma = -1$



$$Y \sim \sigma \lambdabar$$

K. Y. Bliokh et al., PRL (2008)

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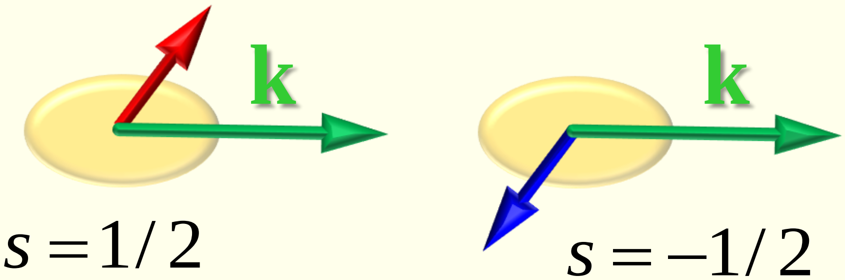
Angular momentum of electron

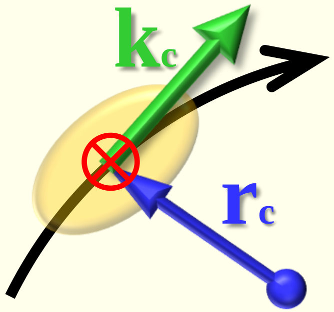
1. Spin AM



$$\mathbf{S} = s\,\mathbf{e}_z$$

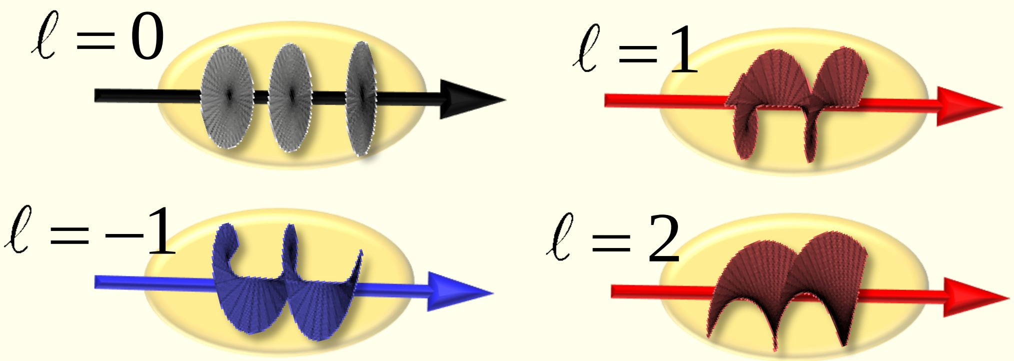
'non-relativistic'

2. Extrinsic orbital AM



$$\mathbf{L}_{ext} = \mathbf{r}_c \times \mathbf{k}_c$$

3. Intrinsic orbital AM ?



$$\mathbf{L}_{int} = \ell\,\boldsymbol{\kappa}$$

'relativistic'

Scalar electron vortex beams

190404 (2007) PHYSICAL REVIEW LETTERS week
9 NOVEM

Semiclassical Dynamics of Electron Wave Packet States with Phase Vortices

Konstantin Yu. Bliokh,[1,2] Yury P. Bliokh,[1,3] Sergey Savel'ev,[1,4] and Franco Nori[1,5]

We consider semiclassical higher-order wave packet solutions of the Schrödinger equation with phase vortices. The vortex line is aligned with the propagation direction, and the wave packet carries a well-defined orbital angular momentum (OAM) $\hbar l$ (l is the vortex strength) along its main linear momentum. The probability current coils around the momentum in such OAM states of electrons. In an electric field, these states evolve like massless particles with spin l. The magnetic-monopole Berry curvature appears in

[1] P. A. M. Dirac, Proc. R. Soc. A **133**, 60 (1931).
[2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959);

vortices appear naturally in t
atoms, quantum Hall fluids, supermedia, ferromagnets, Bose-Einstein condensates, and classical wave fields (e.g., in optics). While 2D vortices in condensed matter physics are pointlike objects, with vorticity being orthogonal to the plane of motion [6], the optical vortices are mainly considered as linear objects in 3D space, with vorticity being aligned with the wave momentum. Wave

Scalar electron vortex beams

$$\left(i\hbar\frac{\partial}{\partial\tau}+\frac{\hbar^2}{2m}\nabla_\perp^2\right)u=0. \qquad (1)$$

$$u_{l,\mathsf{m},\mathsf{n}}(r,\varphi,\zeta,\tau)=u^{\mathrm{LG}}_{l,\mathsf{m}}(r,\varphi,\tau)\,u^{\mathrm{HG}}_{\mathsf{n}}(\zeta). \qquad (2)$$

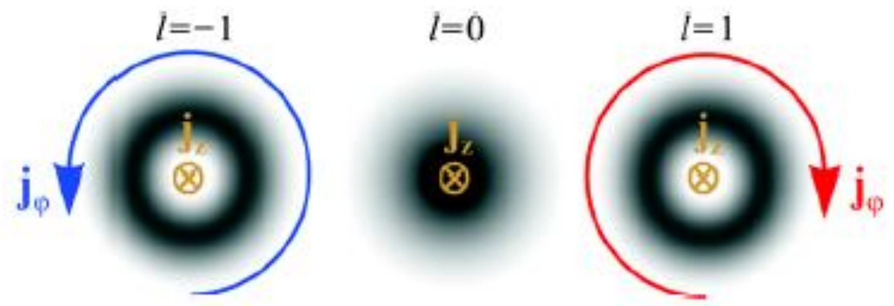


FIG. 1 (color online). Transverse distribution of the probability density ρ in LG beams with $\mathsf{m}=0$ and different values of OAM, l. Shown are the directions of common z component and different φ components of the probability current $\mathbf{j}$.

Scalar electron vortex beams

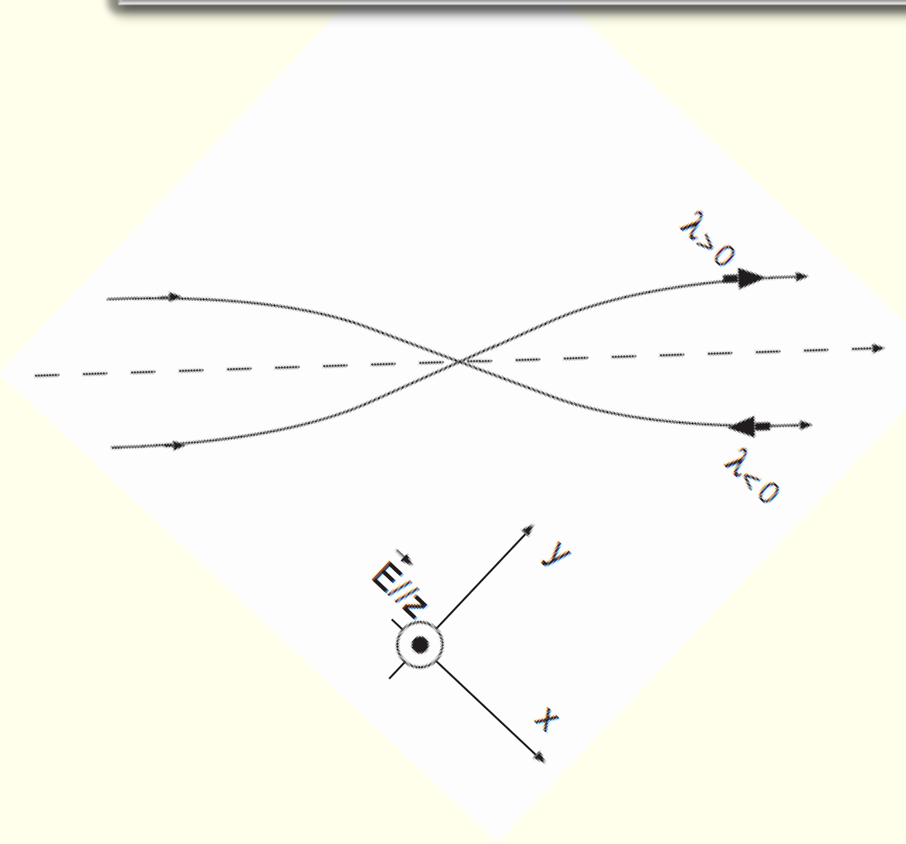
Optics	Electrons
$\mathbf{E}$	ψ
n, ∇n	Φ, $\mathbf{E}$
...	$\mathbf{A}$, $\mathbf{B}$
hologram (grating with dislocation)	crystal plate with dislocation
spiral-thickness lens	spiral-thickness plate
...	magnetic monopole
orbit-orbit interaction: Berry phase and Magnus/Berry force	
Lorentz force, Dirac phase, Zeeman interaction [23], modified density of states	

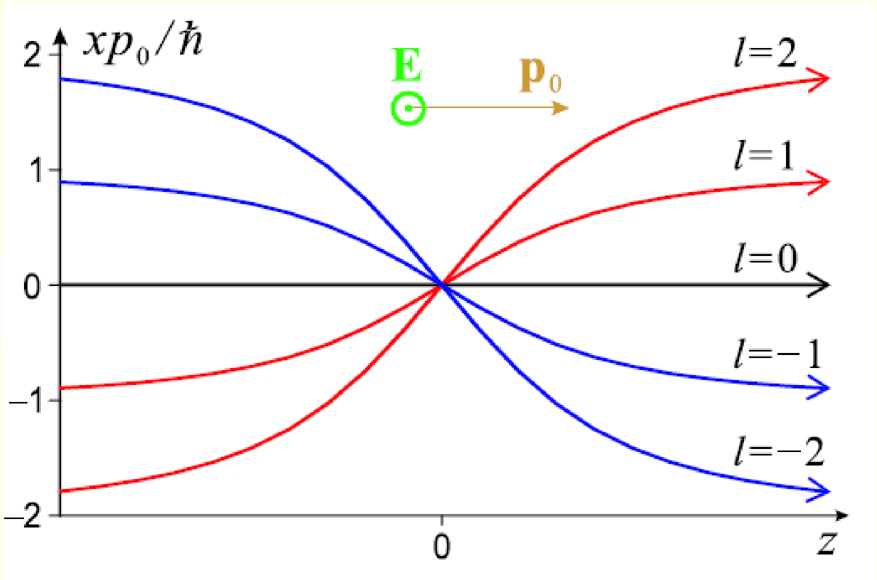
the electron possesses an *intrinsic angular momentum* $\mathbf{L} \equiv \hbar \mathbf{l} = \hbar l \mathbf{e}_z$. The wave packets (2) also have a *magnetic moment* $\boldsymbol{\mu} = g \mu_B \mathbf{l}$ ($\mu_B = e\hbar/2m$, $e = -|e|$, $c = 1$), where $g = 1$ for classical orbital motion, but the g-factor can be different in general (e.g., $g = 2$ for electron spin).

Orbital-Hall effect for electrons

$$\hbar\dot{\mathbf{k}} = e\mathbf{E}\ , \quad \dot{\mathbf{r}} = \hbar\mathbf{k}/m - \ell\mathcal{F}\times\dot{\mathbf{k}}$$

– equations of motion





$$\mathbf{J} = \mathbf{r}\times\mathbf{k} + \ell\,\boldsymbol{\kappa} = const$$

Murakami, Nagaosa, Zhang, Science 2003

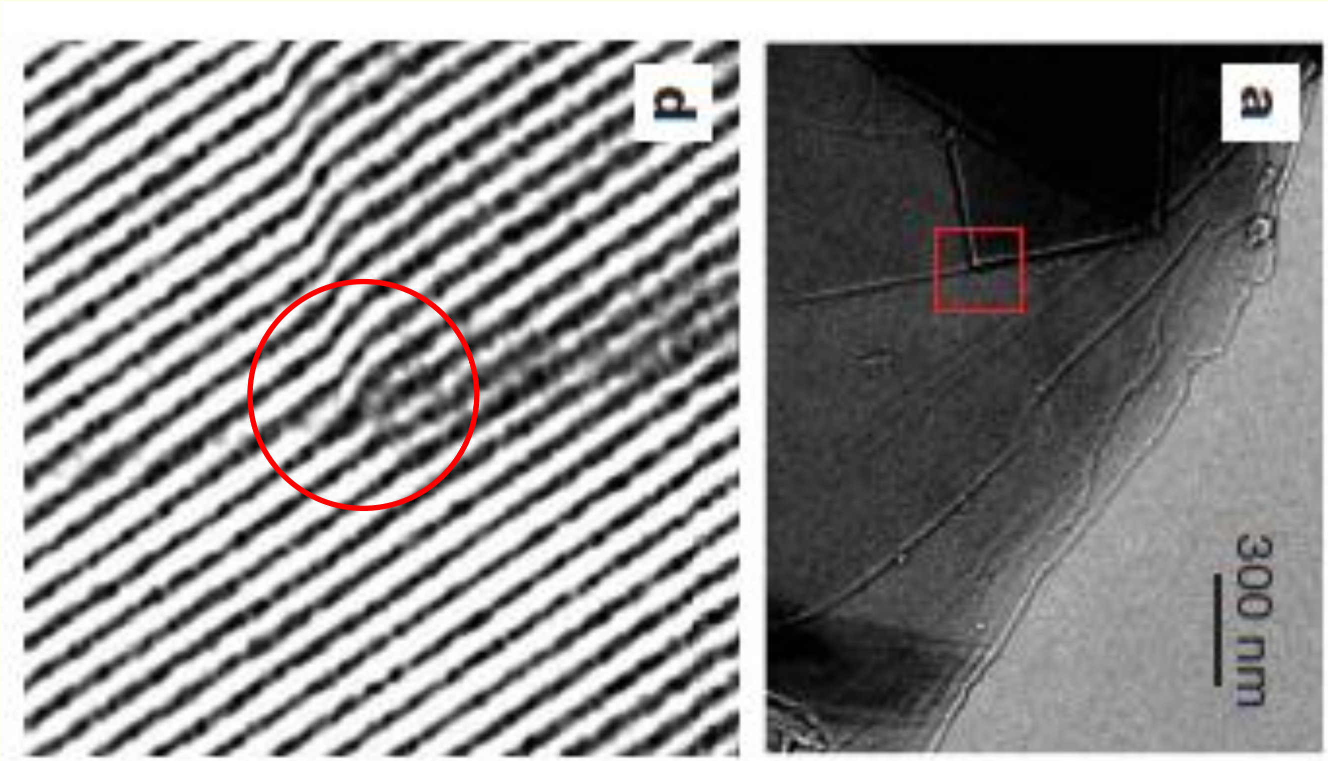
Bliokh et al., PRL 2007

Scalar electron vortex beams

Vol 464 | 1 April 2010 | doi:10.1038/nature08904 nature

Generation of electron beams carrying orbital angular momentum

Masaya Uchida[1] & Akira Tonomura[1]

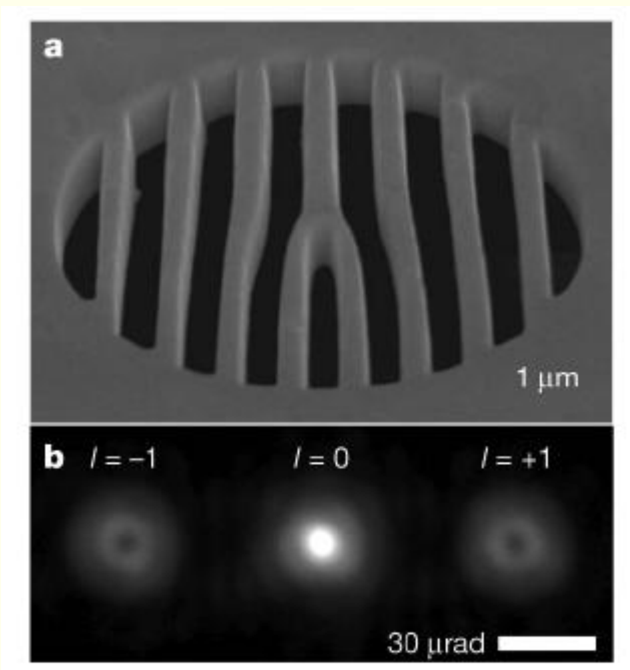


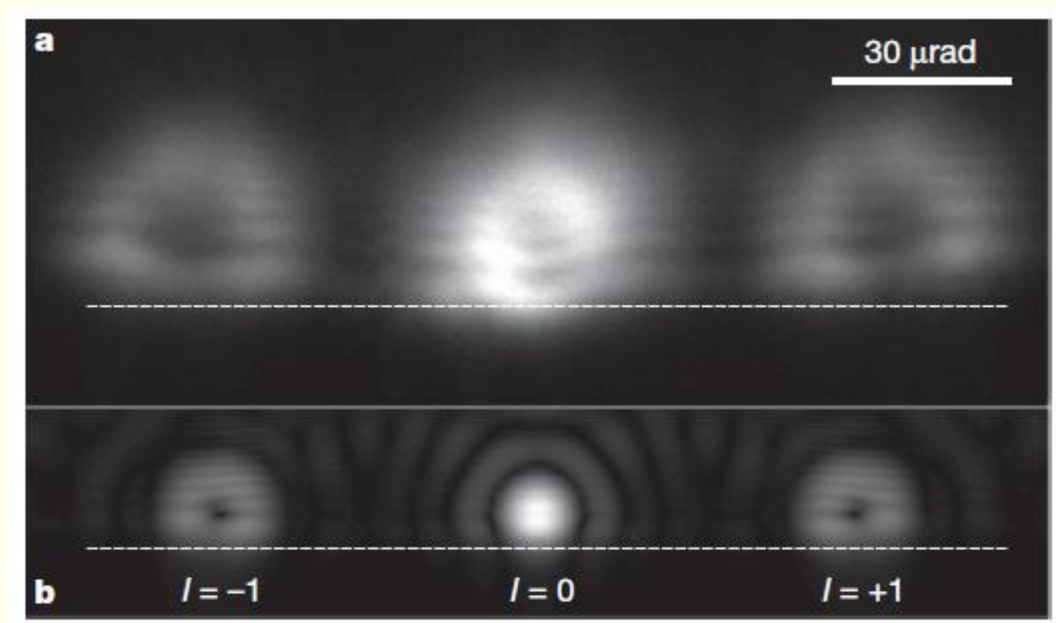
Scalar electron vortex beams

Vol 467 | 16 September 2010 | doi:10.1038/nature09366 nature

Production and application of electron vortex beams

J. Verbeeck[1], H. Tian[1] & P. Schattschneider[2]





Scalar electron vortex beams

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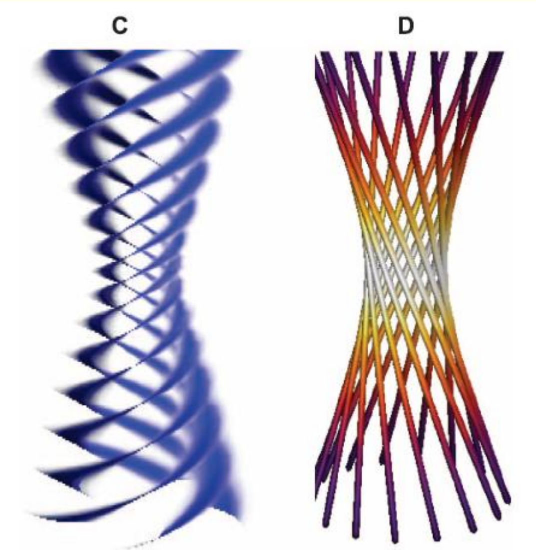
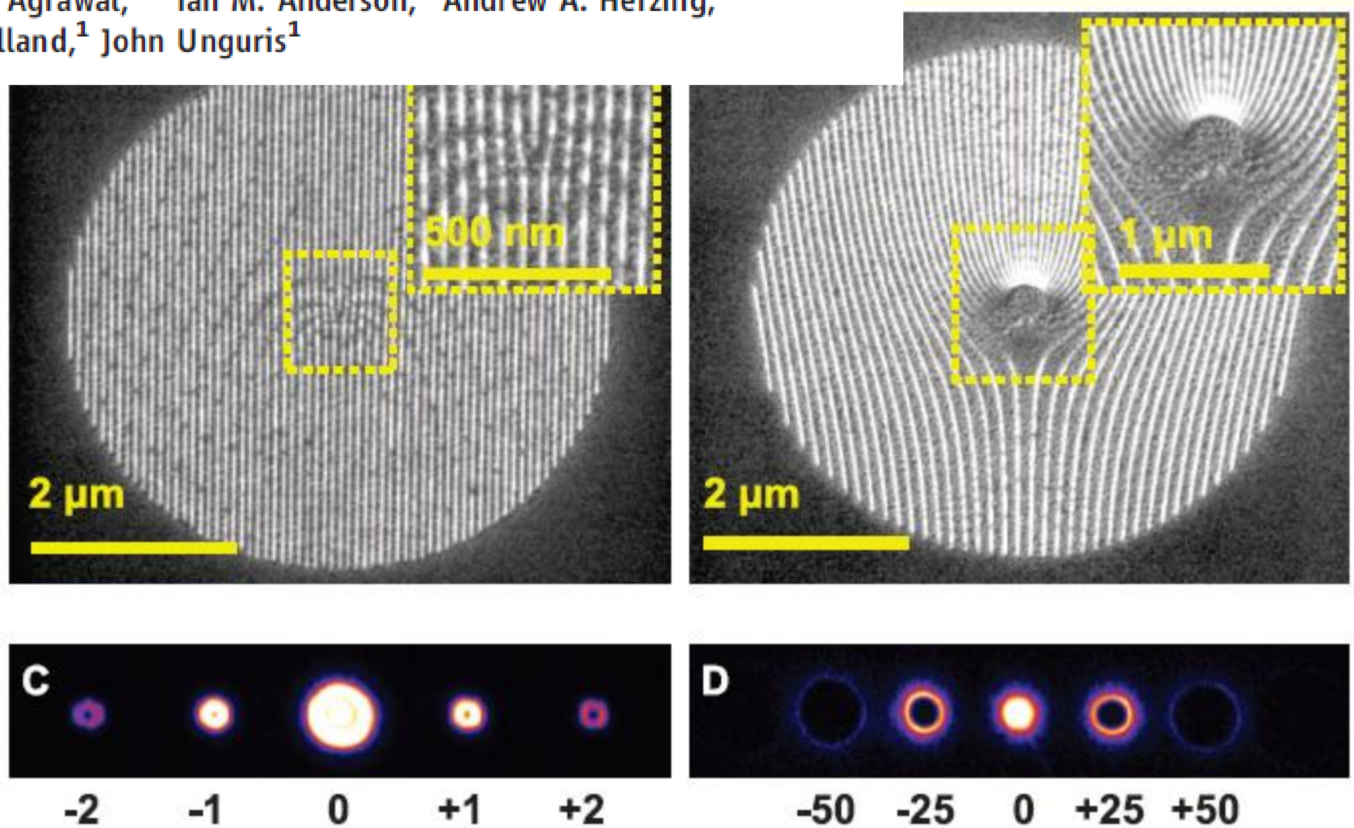
Electron Vortex Beams with High Quanta of Orbital Angular Momentum

Benjamin J. McMorran,[1]* Amit Agrawal,[1,2] Ian M. Anderson,[3] Andrew A. Herzing,[3]
Henri J. Lezec,[1] Jabez J. McClelland,[1] John Unguris[1]

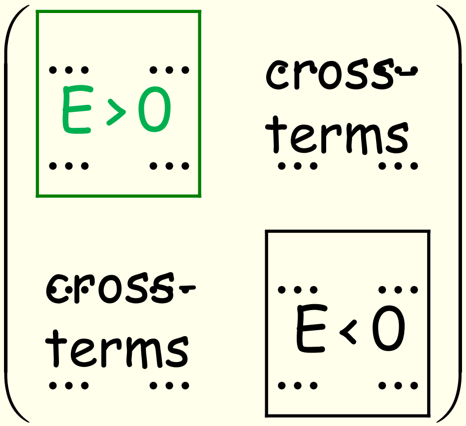




- Basic concepts: Angular momenta, Berry phase, Spin-orbit interactions, Hall effects
- Theory of photon AM
- Application to Bessel beams
- Electron vortex beams
- **Theory of electron AM**

Bessel beams for Dirac electron

$$i\hbar\partial_t\psi = \left(\hat{\boldsymbol{\alpha}}\cdot\hat{\mathbf{p}} + \hat{\beta}\, m\right)\psi$$ – Dirac equation:



$$\psi_p = W(\mathbf{p})\, e^{i\hbar^{-1}(\mathbf{p}\cdot\mathbf{r} - E\,t)}$$ – plane wave

$$W = \frac{1}{\sqrt{2E}}\begin{pmatrix} \sqrt{E+m}\, w \\ \sqrt{E-m}\,\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{\kappa}\, w \end{pmatrix}$$ – polarization bi-spinor

$$w = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$ – spinor in the rest frame

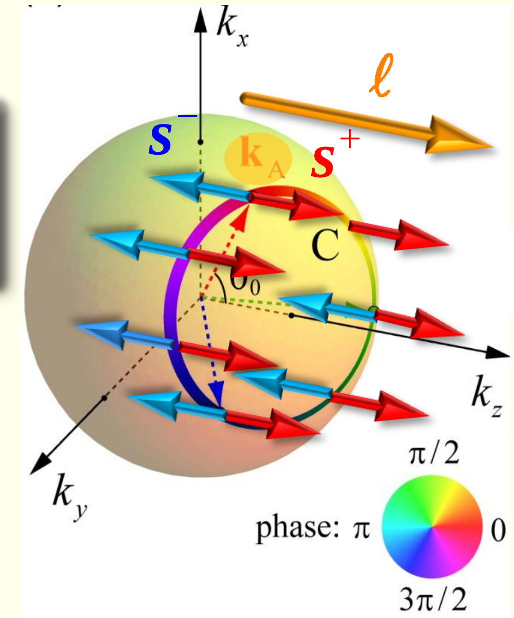
$$w^{\pm} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to S_z = \frac{1}{2}, -\frac{1}{2}$$

Bessel beams for Dirac electron

$$\tilde{\psi}_\ell^s(\mathbf{p}) \propto W^s \delta(\theta-\theta_0) e^{i\ell\phi}$$ – spectrum

$$\rho_\ell^s(r) = [1-\Delta/2] J_\ell^2(\tilde{r}) + \Delta J_{\ell+2s}^2(\tilde{r})/2$$

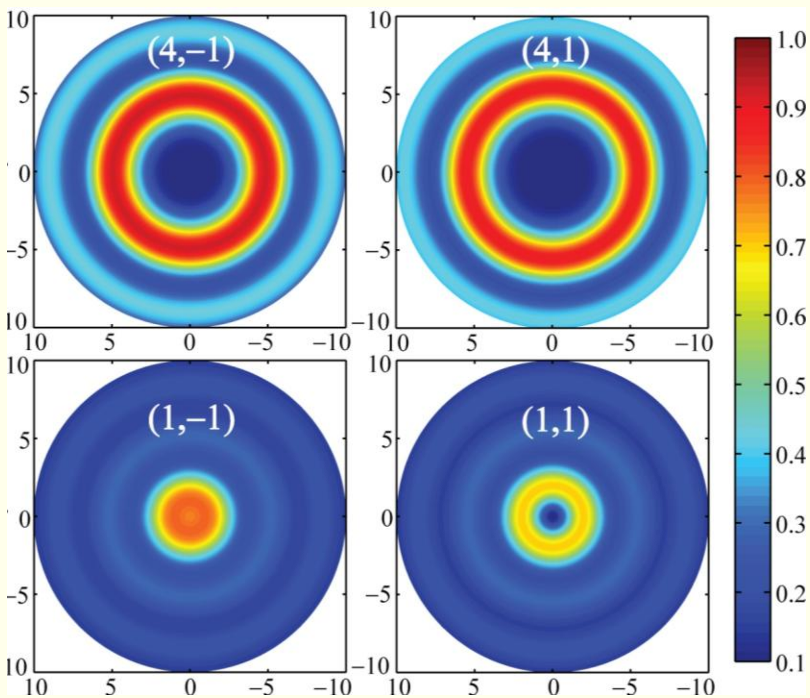
– spin-dependent intensity



$s = -1/2$ $s = 1/2$

$\ell = 4$

$\ell = 1$



$$\Delta = \left(1-\frac{m}{E}\right)\sin^2\theta_0 = \Phi$$

– SOI strength

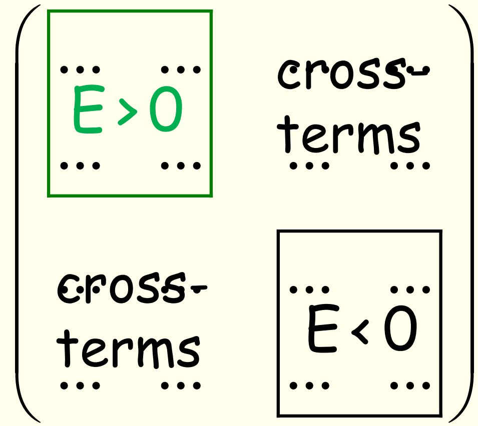
Bliokh, Dennis, Nori, arXiv:1105.0331

Angular momentum

$\hat{U}_{FW}(\mathbf{p})$ – Foldy-Wouthhuysen transformation

$\hat{\mathcal{P}}^{+}$ – projector onto E>0 subspace

$$\hat{\mathbf{O}}' = \hat{\mathcal{P}}^{+}\left(\hat{U}^{\dagger}_{FW}\hat{\mathbf{O}}\hat{U}_{FW}\right):$$



$$\hat{\mathbf{S}}' = \hat{\mathbf{S}} - \hat{\mathbf{\Delta}}, \quad \hat{\mathbf{L}}' = \hat{\mathbf{L}} + \hat{\mathbf{\Delta}}$$

$$\hat{\mathbf{r}}' = i\partial_{\mathbf{k}} - \hbar\hat{\mathcal{A}}$$

$$\hat{\mathbf{\Delta}} = -\hbar\hat{\mathcal{A}} \times \mathbf{p}$$

$$\hat{\mathcal{A}} = -\frac{\mathbf{p}\times\hat{\boldsymbol{\sigma}}}{2p^2}\left(1 - \frac{m}{E}\right)$$

$$L_z = \ell + s\Delta, \quad S_z = s(1-\Delta)$$

– OAM and SAM for Bessel beams

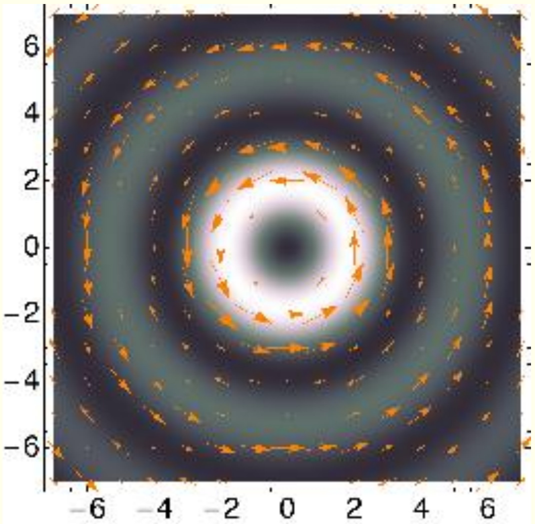
Bliokh, Dennis, Nori, arXiv:1105.0331

Magnetic moment for Dirac electron

$$\mathbf{M} = \frac{e}{2}\int \mathbf{r} \times \mathbf{j}\, dV$$

$$\mathbf{j} = \psi^{\dagger}\hat{\boldsymbol{\alpha}}\psi = \mathbf{p}/E$$

– magnetic moment from current



$$\mathbf{M} = \langle\psi|\hat{\mathbf{M}}|\psi\rangle, \quad \hat{\mathbf{M}} = \frac{e}{2}\hat{\mathbf{r}} \times \hat{\boldsymbol{\alpha}}$$

– operator !

$$\hat{\mathbf{M}}' = \hat{\mathcal{P}}^{+}\left(\hat{U}_{FW}^{\dagger}\hat{\mathbf{M}}\hat{U}_{FW}\right) = \frac{e\hbar}{2E}\left(\hat{\mathbf{L}}' + 2\hat{\mathbf{S}}'\right)$$

$\Rightarrow$

$$\mathbf{M} = \frac{e\hbar}{2E}\left(\ell + 2s - \Delta\right)$$

– final result

slightly differs from Gosselin et al. (2008); Chuu, Chang, Niu, (2010)

- Spin and orbital AM of light, geometric phase, spin- and orbital-Hall effects of light
- General theory for nonparaxial optical fields: modified SAM, OAM, and position operators.
- Bessel-beams example, spin-orbit splitting of caustics and Hall effects
- Electron vortex beams, AM of electron, orbital-Hall effect.
- Relativistic nonparaxial electron: Bessel beams from Dirac equation
- General theory of SAM, OAM, position, and magnetic moment of Dirac electron.

THANK YOU!

Position operator of photon

Berry phase

Spin and Orbital AM of photons

Spin-orbit interaction of light:
(i) Hall effects
(ii) AM conversion