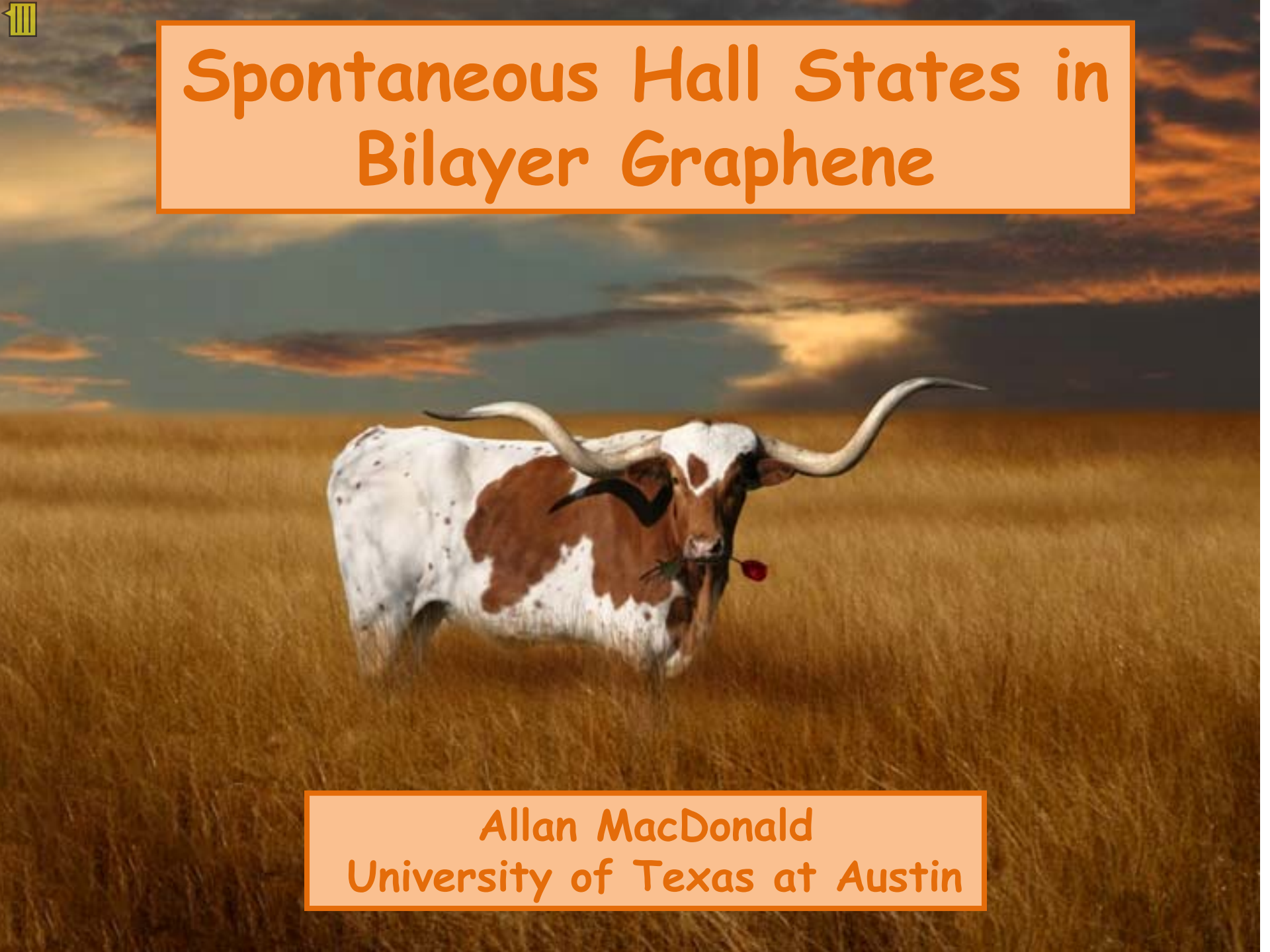
Spontaneous Hall States in Bilayer Graphene

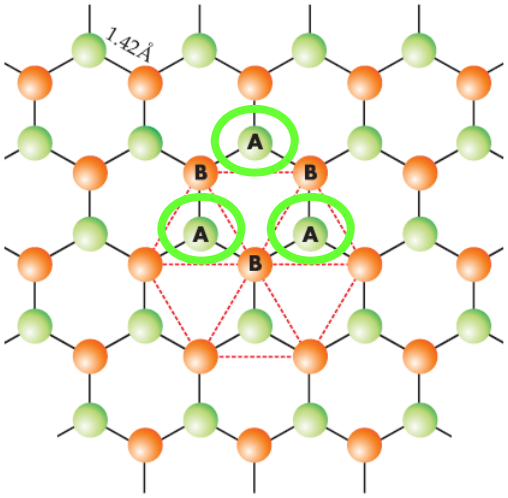


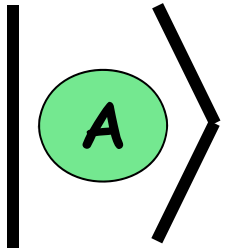
Allan MacDonald
University of Texas at Austin

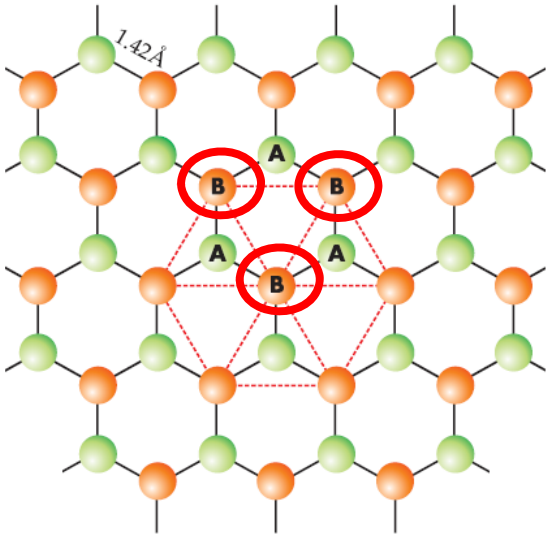

ARMY RESEARCH OFFICE
Jason
Hongki
Jeil
Sahu
Wei-Cheng
NRI
Reza
Yafis
Office of Basic Energy Sciences
Serving the Present, Shaping the Future
Marco
Kentaro Nomura
Department of Physics
University of Texas at Austin
NSF
Nikolai
Tami

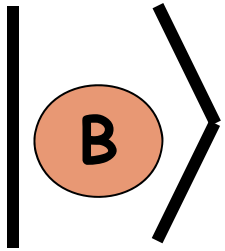
◆ Bilayer Graphene

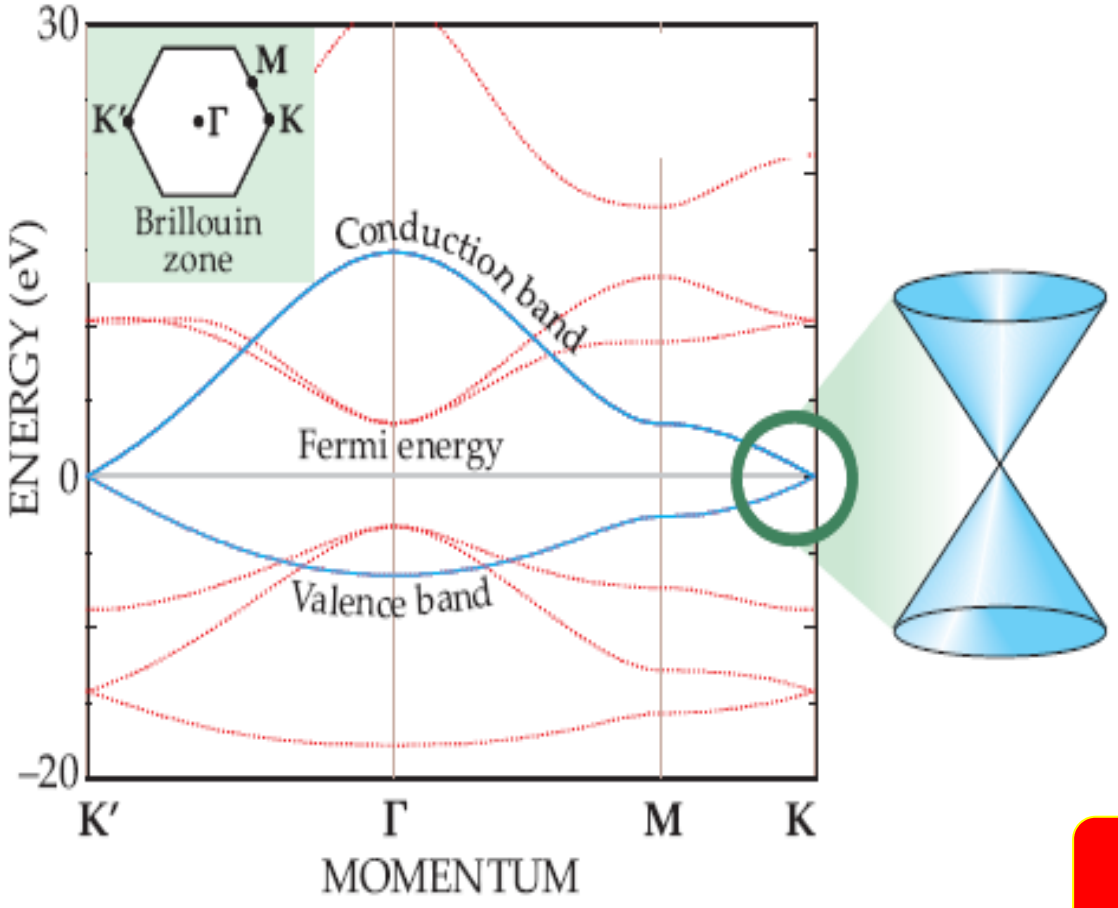
Sublattice-Pseudospins











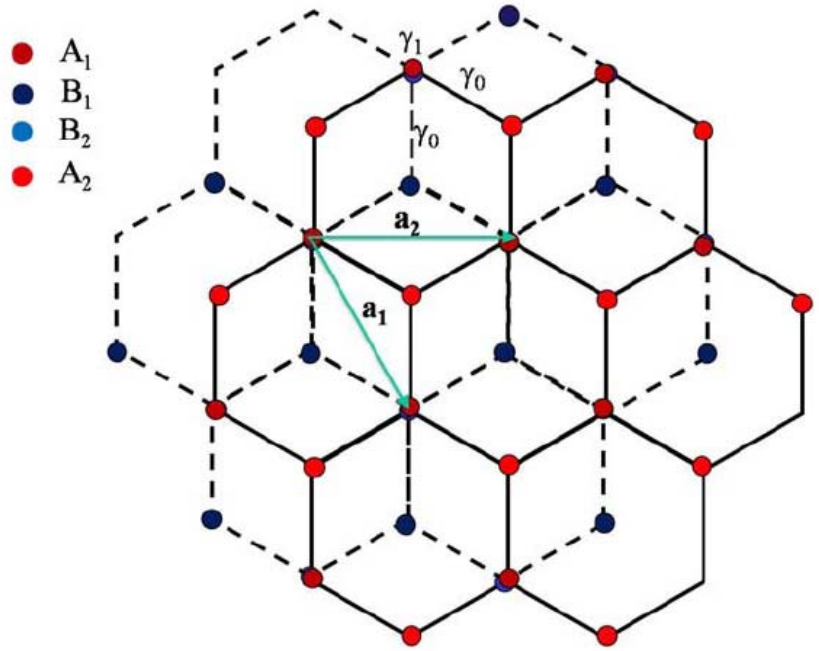
Gapless !

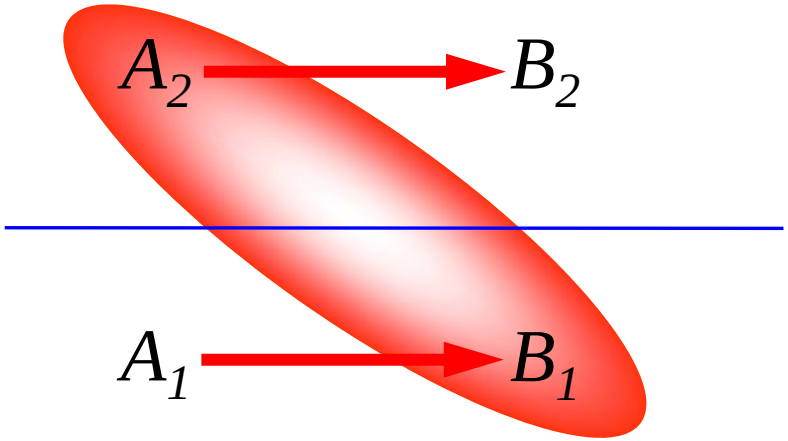
Pseudospin Chirality !

v ~ 100nm/(10^{-13} s)

$$H = -\hbar v \mathbf{k} \cdot \boldsymbol{\tau}$$

Bilayer AB stacking





Novoselov et al. – Nature Phys. (2006)
McCann & Falko – PRL (2006)

$$\hat{\mathcal{H}}_{\text{band}} = -\sum_{k,\sigma',\sigma} \hat{c}^{\dagger}_{\boldsymbol{k},\sigma'} \left\{ \varepsilon_0(k_c) \left(\frac{k}{k_c}\right)^{J} \left[\cos(J\phi_{\boldsymbol{k}})\, \tau^{x}_{\sigma',\sigma} + \sin(J\phi_{\boldsymbol{k}})\, \tau^{y}_{\sigma',\sigma} \right] + \frac{V_g}{2}\, \tau^{z}_{\sigma',\sigma} \right\} \hat{c}_{\boldsymbol{k},\sigma}$$

±

J=2

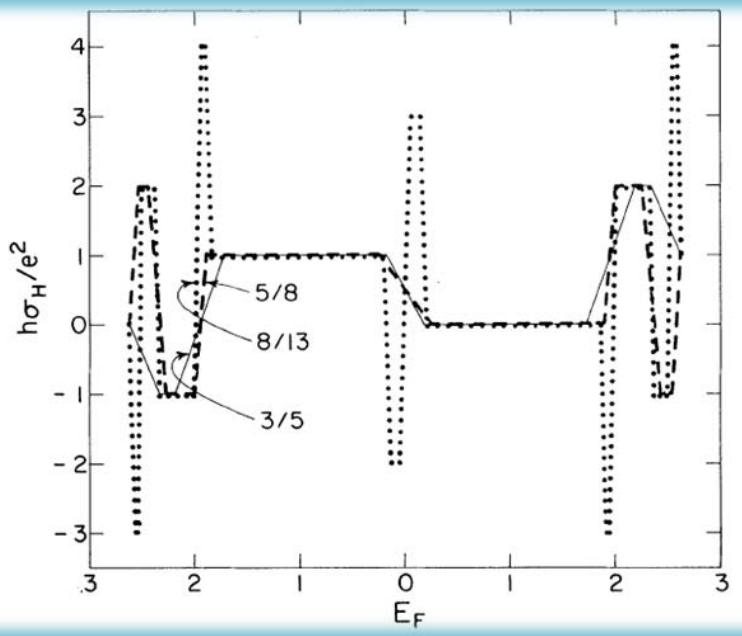
- Quantized Anomalous Hall Effect

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)

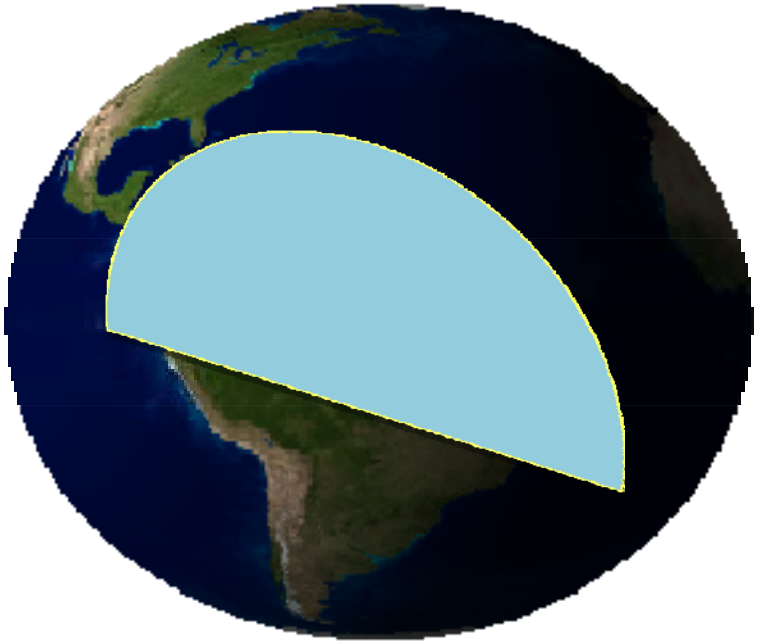
$$\sigma_H = \frac{ie^2}{2\pi h} \sum \int d^2k \int d^2r \left(\frac{\partial u^*}{\partial k_1} \frac{\partial u}{\partial k_2} - \frac{\partial u^*}{\partial k_2} \frac{\partial u}{\partial k_1} \right)$$

Berry Curvature
Chern Index



Square Lattice
nearest neighbor
tight-binding
model

Pseudospin-1/2 Berry Curvature



$$n = \frac{1}{4\pi} \int d^2\mathbf{k} (\partial_{k_x} \hat{h} \times \partial_{k_y} \hat{h}) \cdot \hat{h}.$$

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

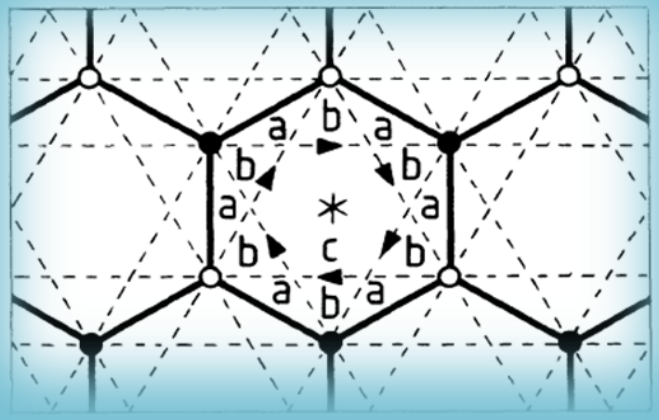
Department of Physics, University of California, San Diego, La Jolla, California 92093

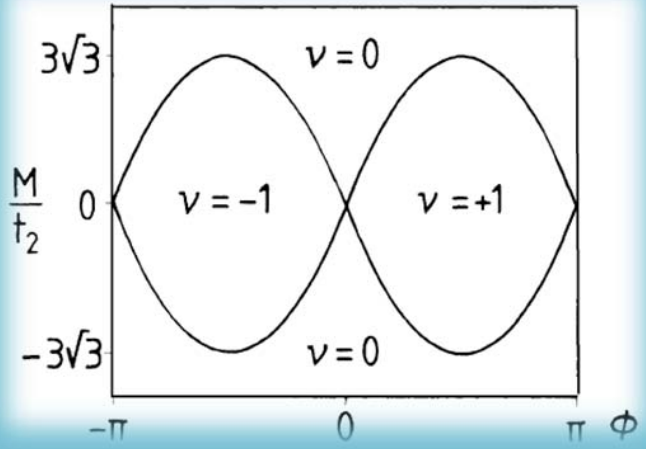
(Received 16 September 1987)

$$H(\mathbf{k}) = 2t_2\cos\phi\left(\sum_i \cos(\mathbf{k}\cdot\mathbf{b}_i)\right)\mathbf{I} + t_1\left(\sum_i [\cos(\mathbf{k}\cdot\mathbf{a}_i)\sigma^1 + \sin(\mathbf{k}\cdot\mathbf{a}_i)\sigma^2]\right) + \left[M - 2t_2\sin\phi\left(\sum_i \sin(\mathbf{k}\cdot\mathbf{b}_i)\right)\right]\sigma^3$$

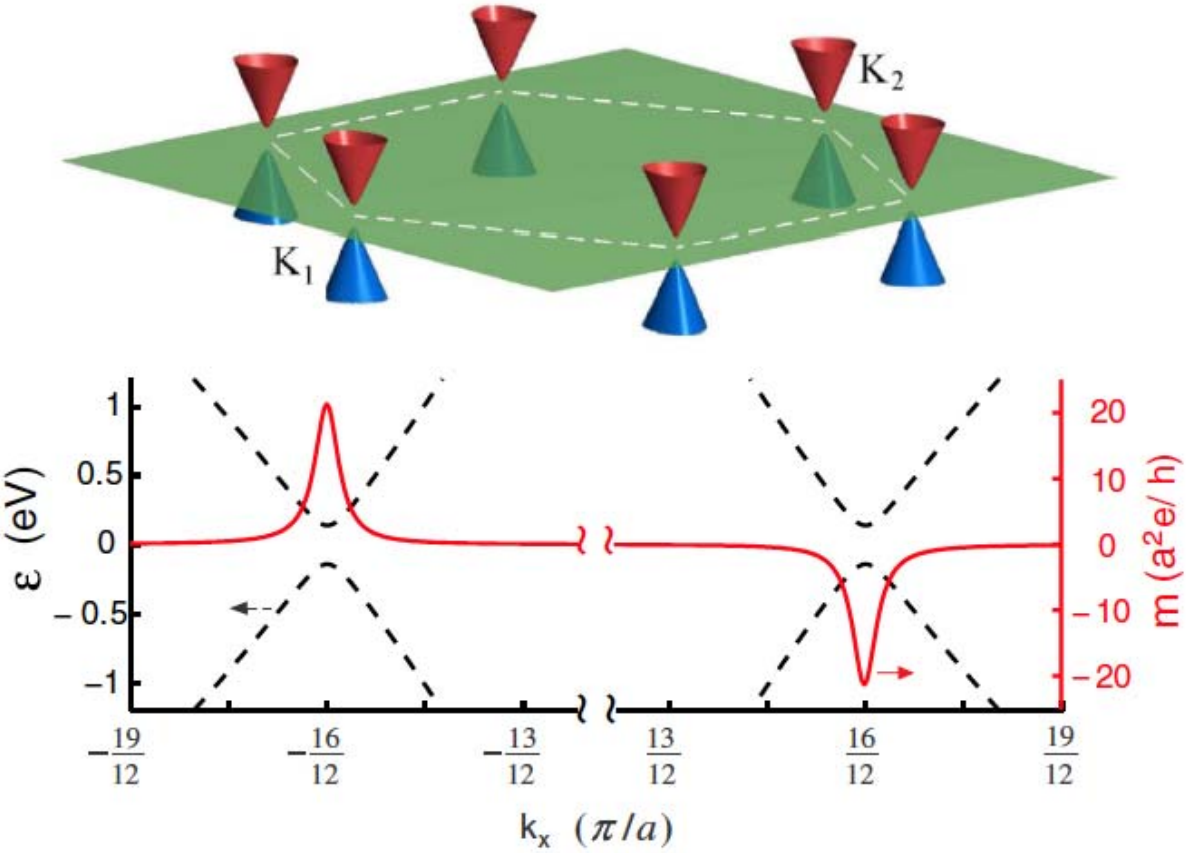
Chern Index Phase Diagram

Pseudospin-Orbit Coupling





Bilayer Graphene Valley Hall Effect

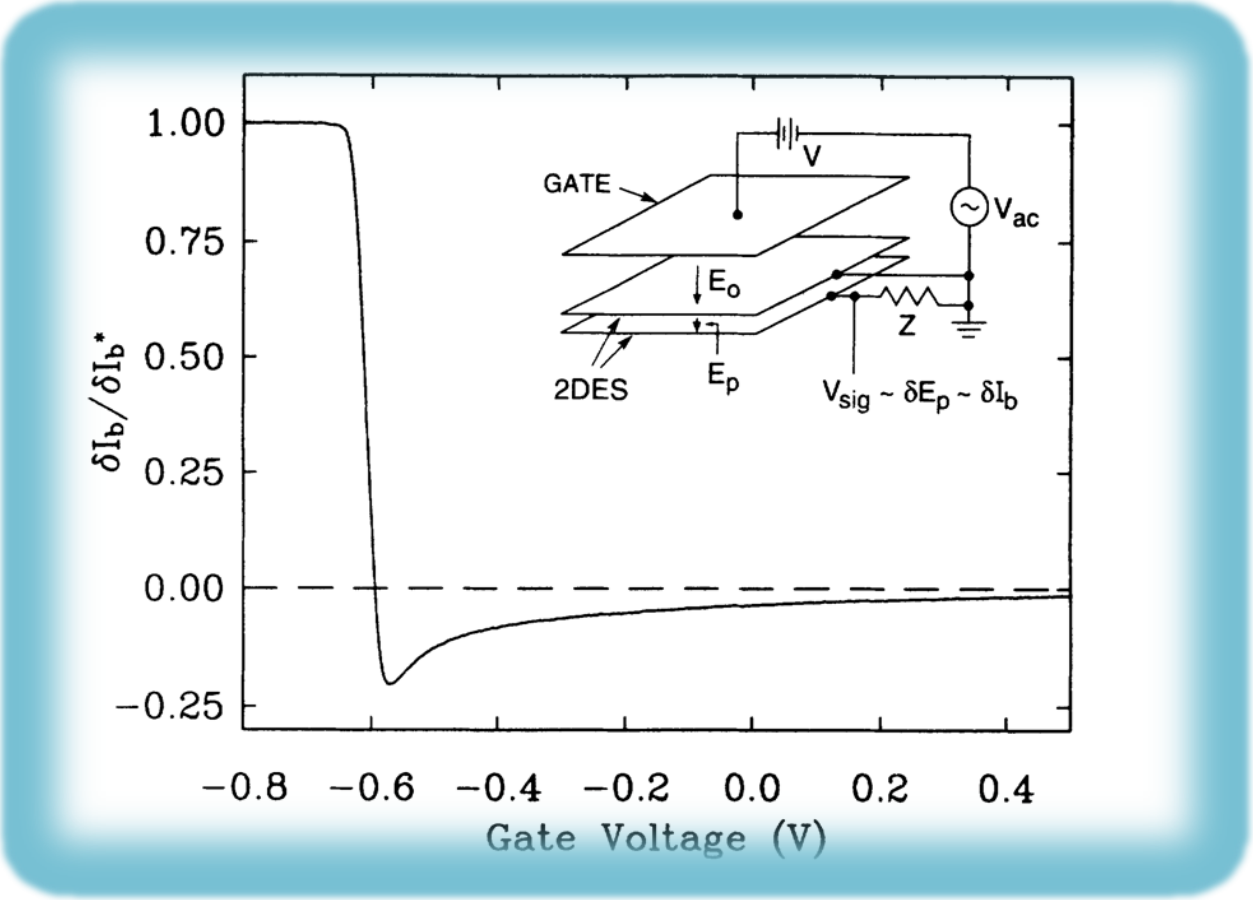


Di Xiao et al., PRL (2008)

◆ Spontaneous Quantum Hall Effects in Bilayer Graphene

Compressibility of the two-dimensional electron gas: Measurements of the zero-field exchange energy and fractional quantum Hall gap

J. P. Eisenstein, L. N. Pfeiffer, and K. W. West
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 22 December 1993)



Staging transitions in multiple-quantum-well systems

A. H. MacDonald*
National Research Council of Canada, Ottawa K1A 0R6, Canada
(Received 3 August 1987)

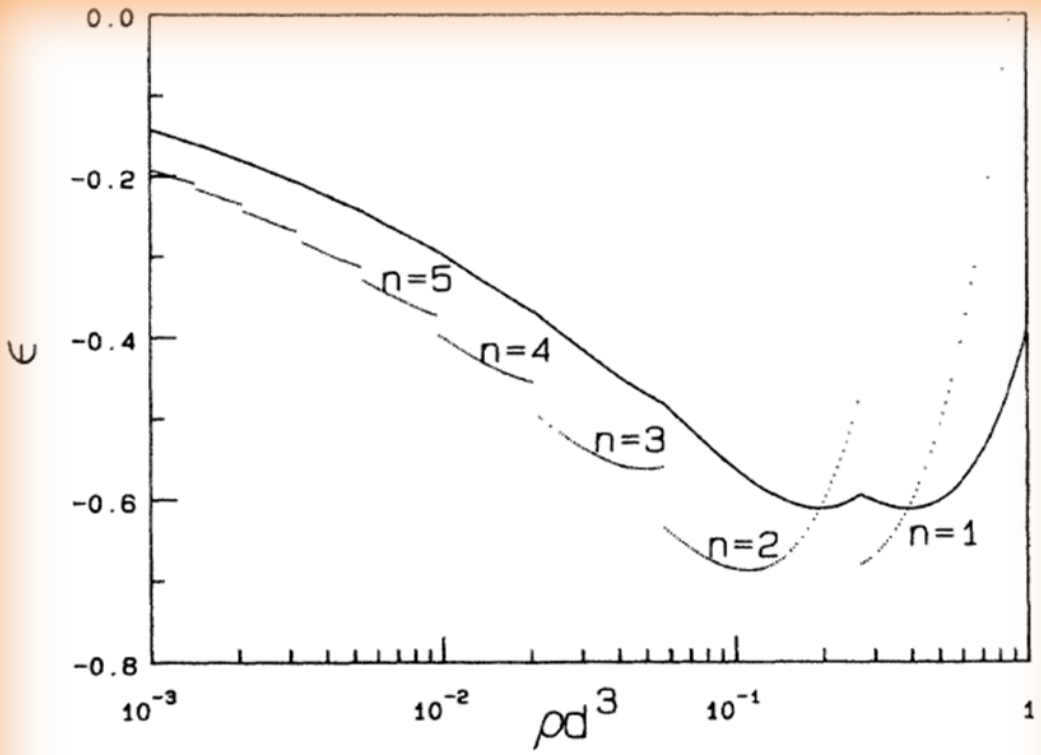
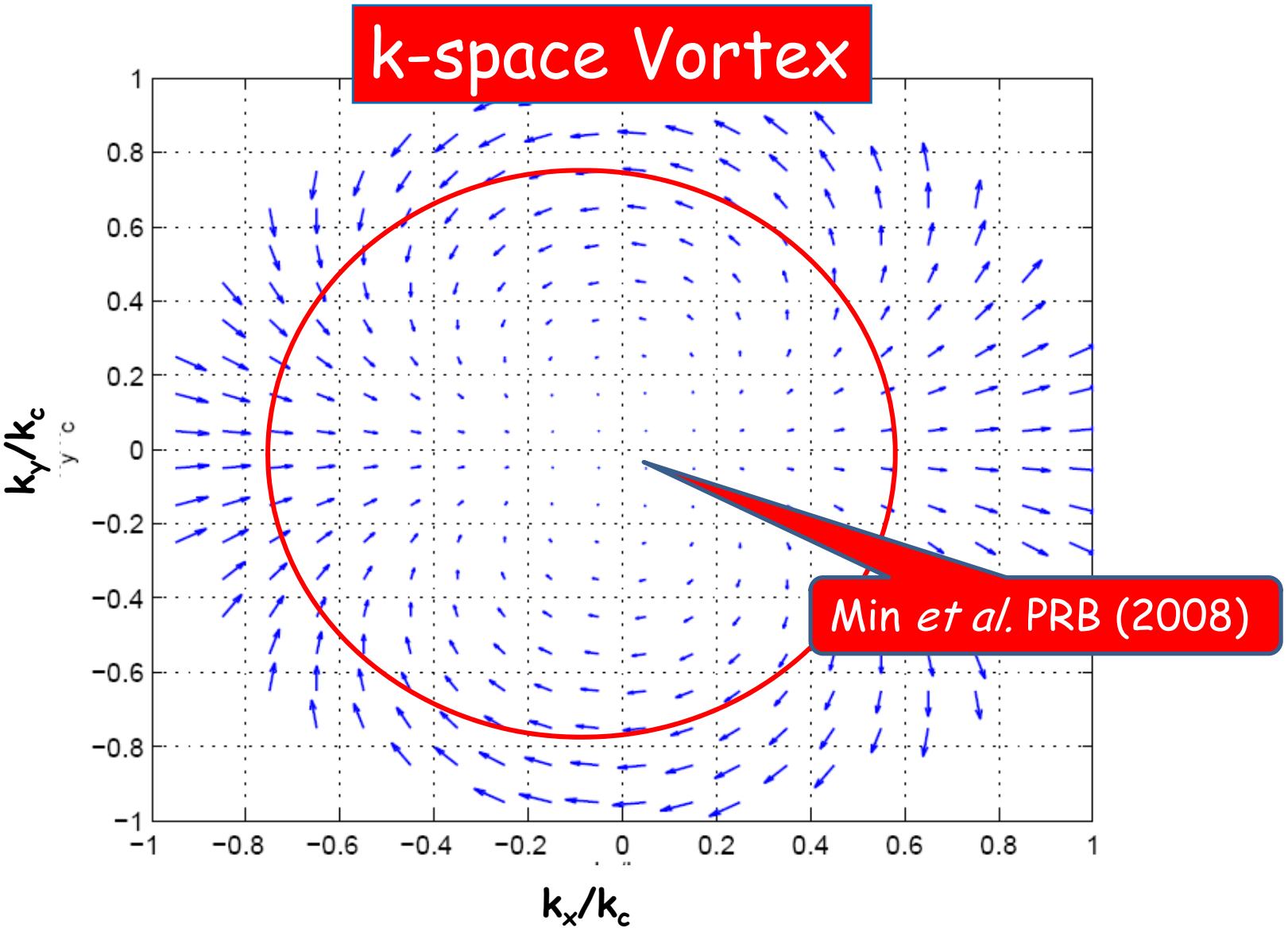
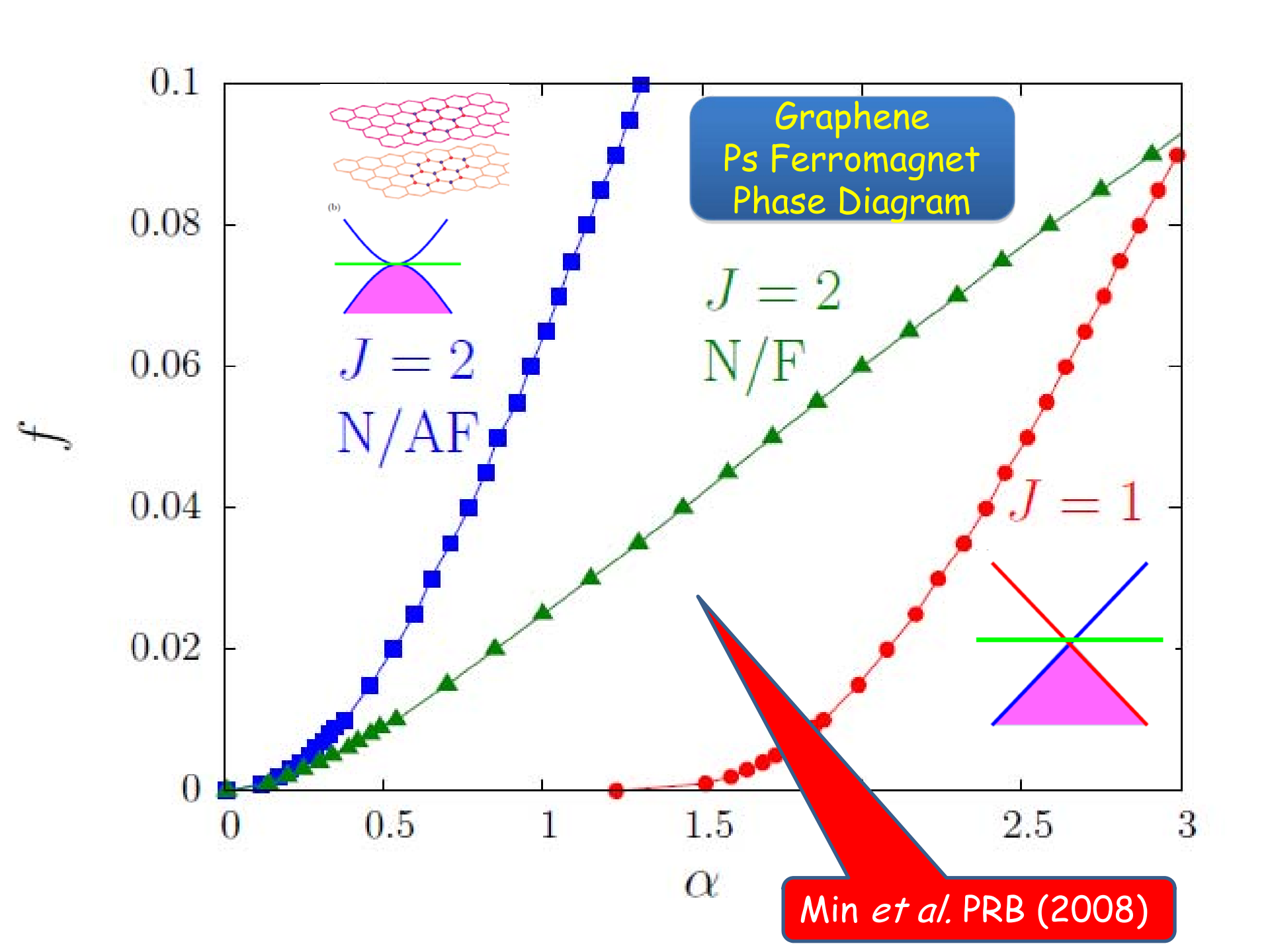


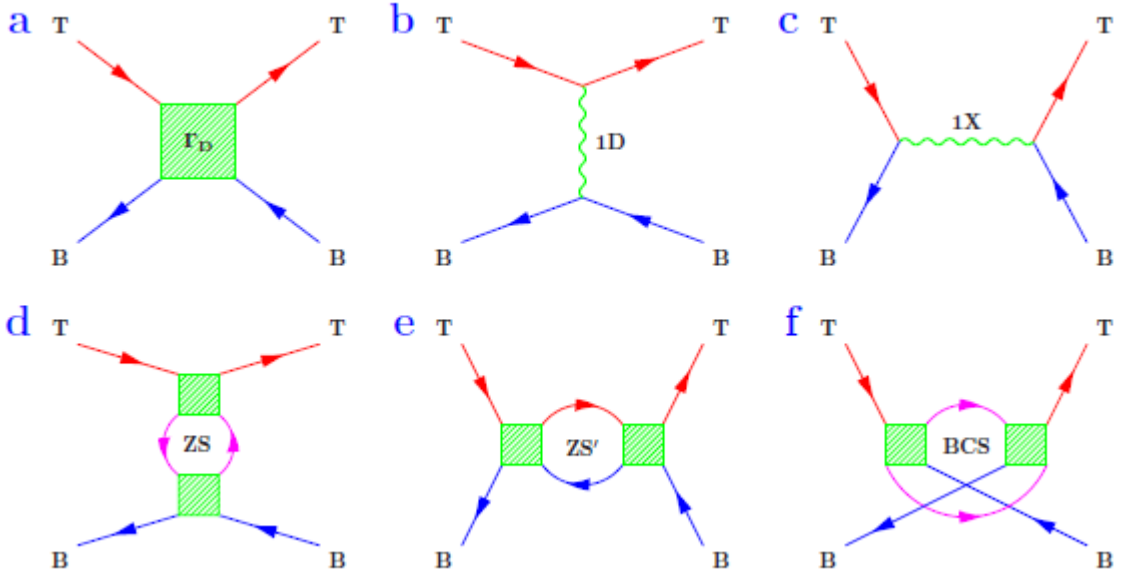
FIG. 1. Energy per electron, ϵ, and chemical potential, μ, in units of $e^2/\epsilon d$ vs $\bar{\rho} d^3$. The solid line (ϵ) is continuous while the dotted line (μ) is discontinuous at each staging transition.

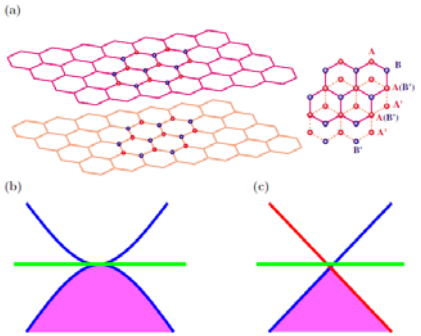
Bilayer Graphene Ps Ferromagnet




Graphene
Ps Ferromagnet
Phase Diagram
$J = 2$
N/AF
$J = 2$
N/F
$J = 1$
f
α
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2.5
3
Min et al. PRB (2008)

Beyond Mean-Field Theory





Diagrams	ZS	ZS′	BCS	One-loop
1DES	0	$u^2 \ln(s)$	$-u^2 \ln(s)$	0
Graphene bilayer	$\frac{1}{2}\Gamma_D^2 \ln(s)$	$\frac{1}{2}\Gamma_D^2 \ln(s)$	0	$\Gamma_D^2 \ln(s)$

Fan *et al.* PRB (2010) Sun *et al.* PRL (2009)
Vafek & Yang, PRB (2010) Lemonik et al. PRB (2010)

Quasiparticle Hamiltonian

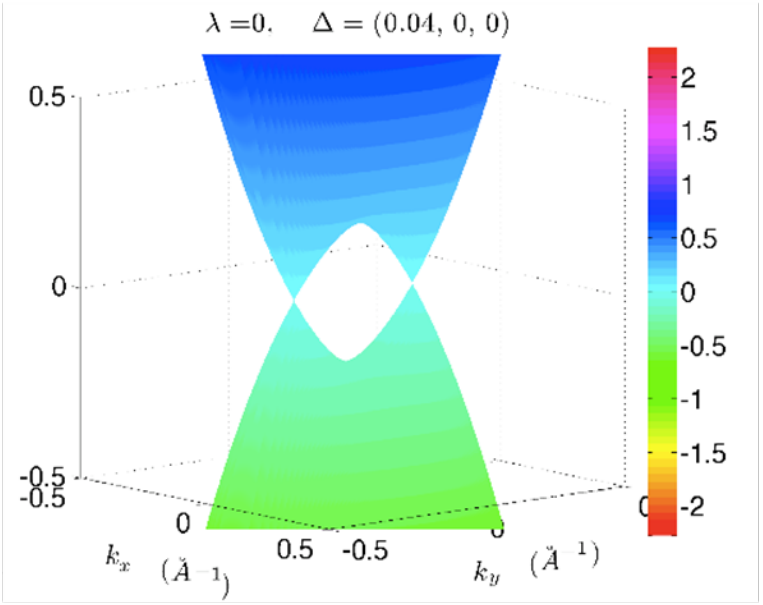
$$\mathcal{H}(\vec{k}) = -[\vec{B}_{\rm band}(\vec{k}) + \vec{B}_{\rm int}(\vec{k})] \cdot \vec{\tau}$$

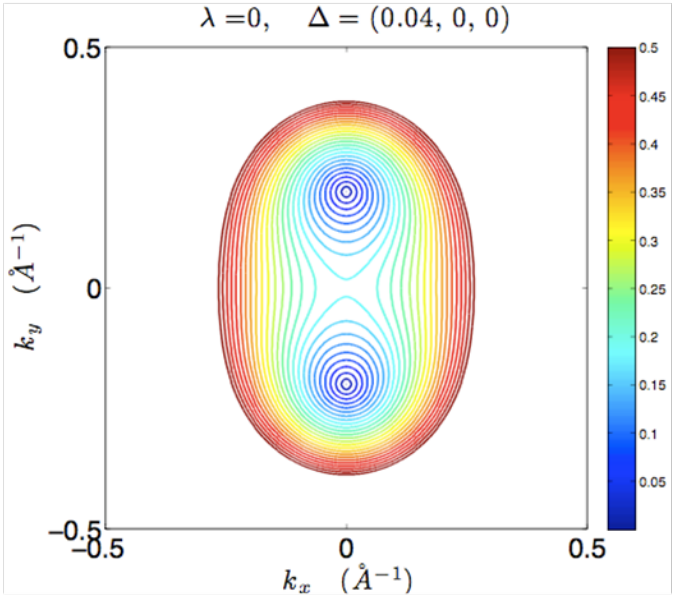
$$\vec{B}_{\rm band}(\vec{k}) = \frac{\hbar^2 k^2}{2m^*} \left(\cos(2\phi_{\vec{k}}), \sin(2\phi_{\vec{k}}), 0\right)$$

Pseudospin Polarization

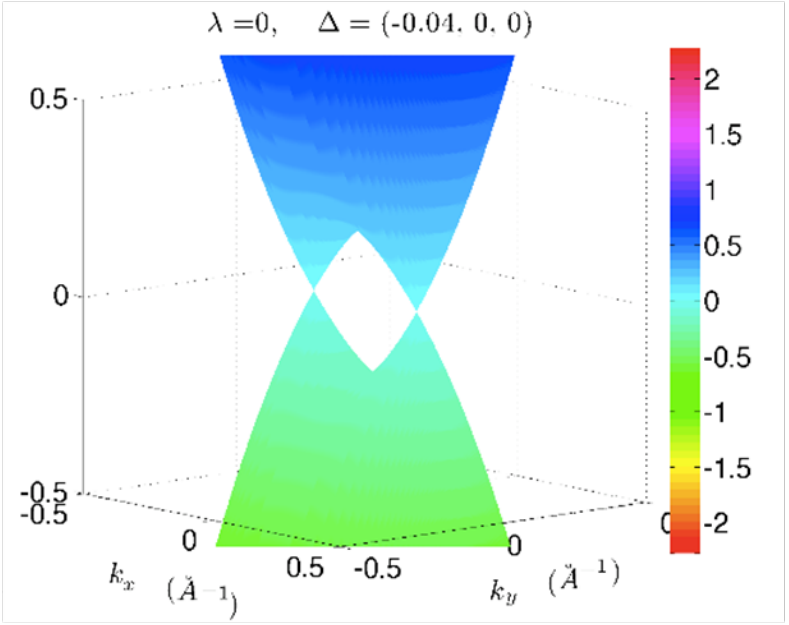
$$\vec{B}_{\rm int}(\vec{k}) = \frac{1}{2A} \sum_{\vec{k}'} V(\vec{k} - \vec{k}')\, \hat{n}(\vec{k}')$$

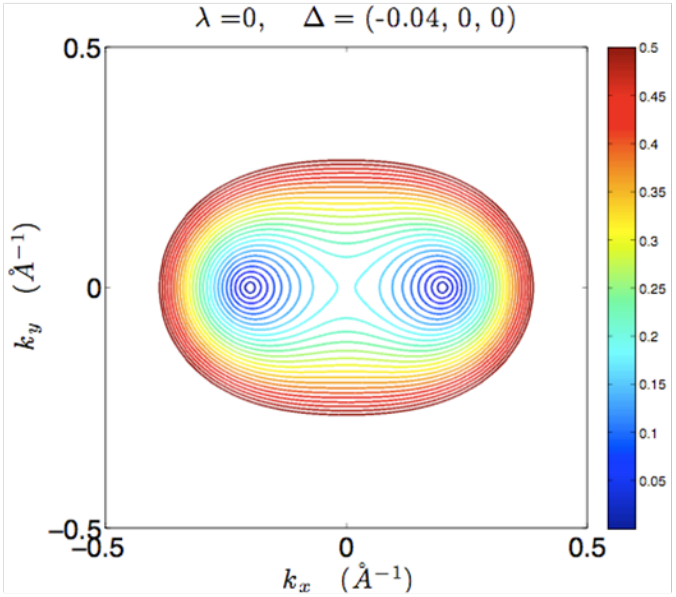
$\Delta_x > 0$



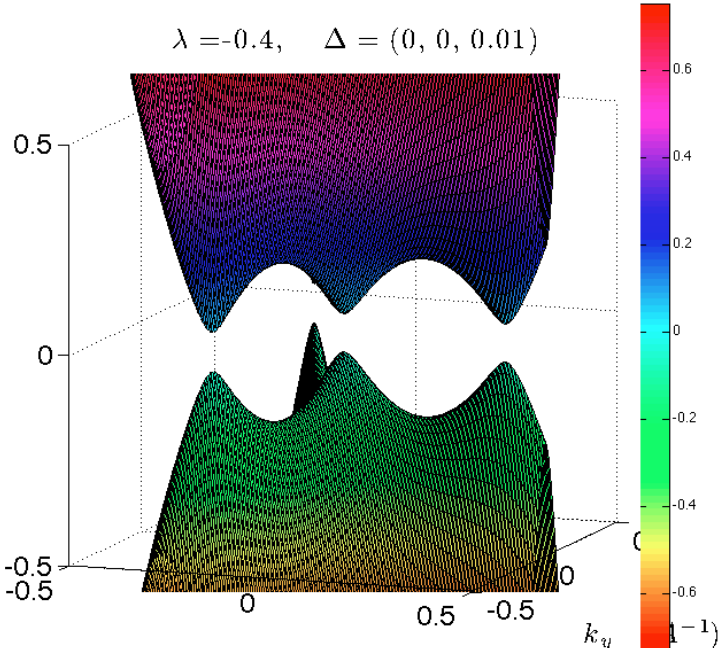


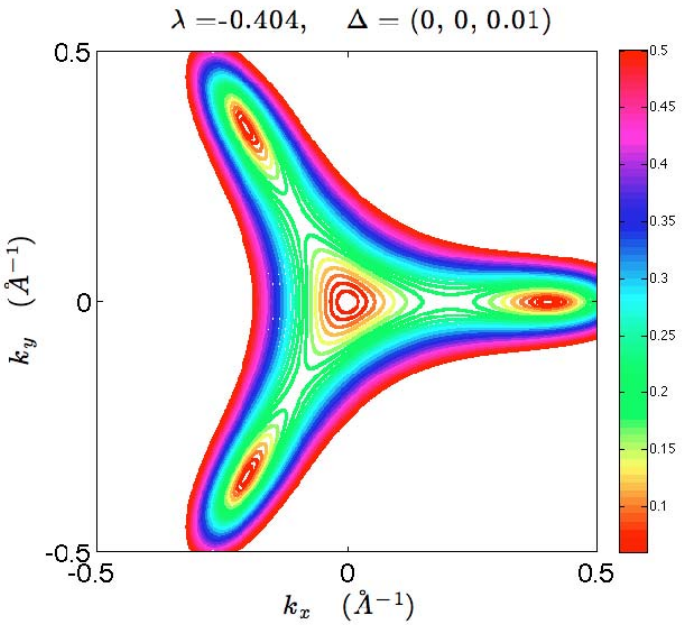
$\Delta_x < 0$



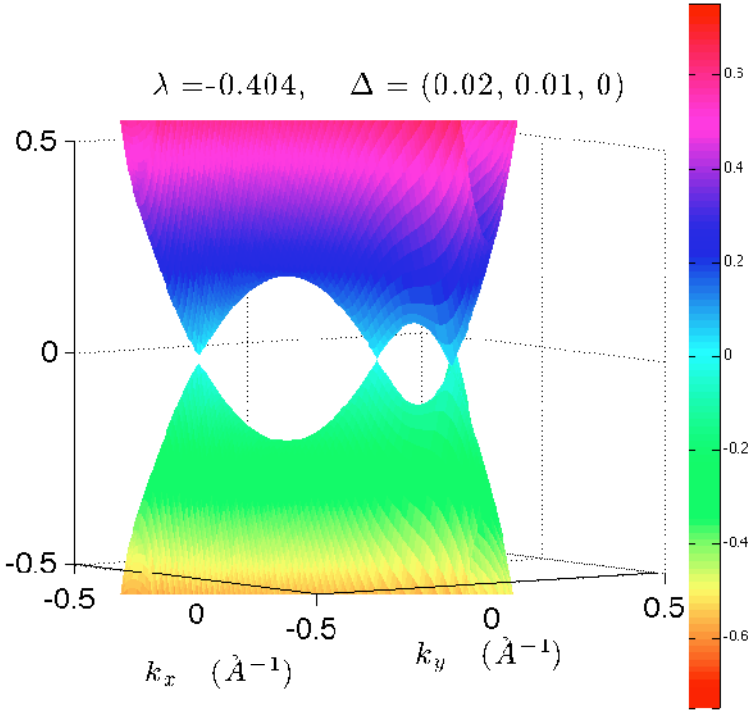


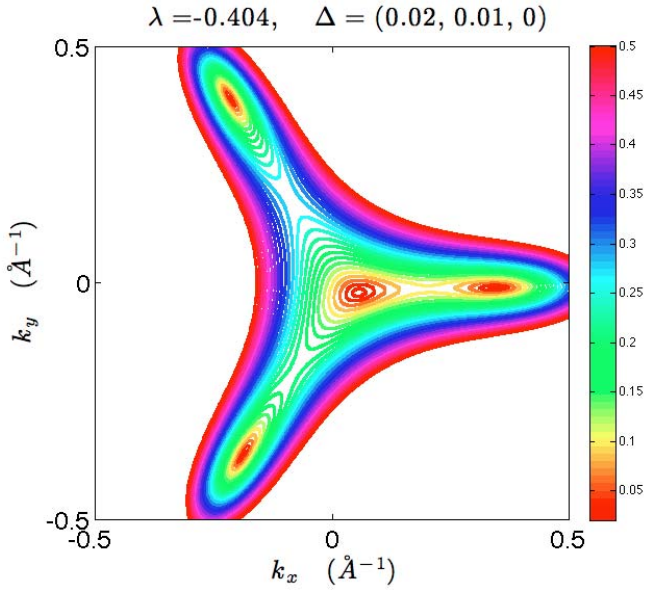
gapped

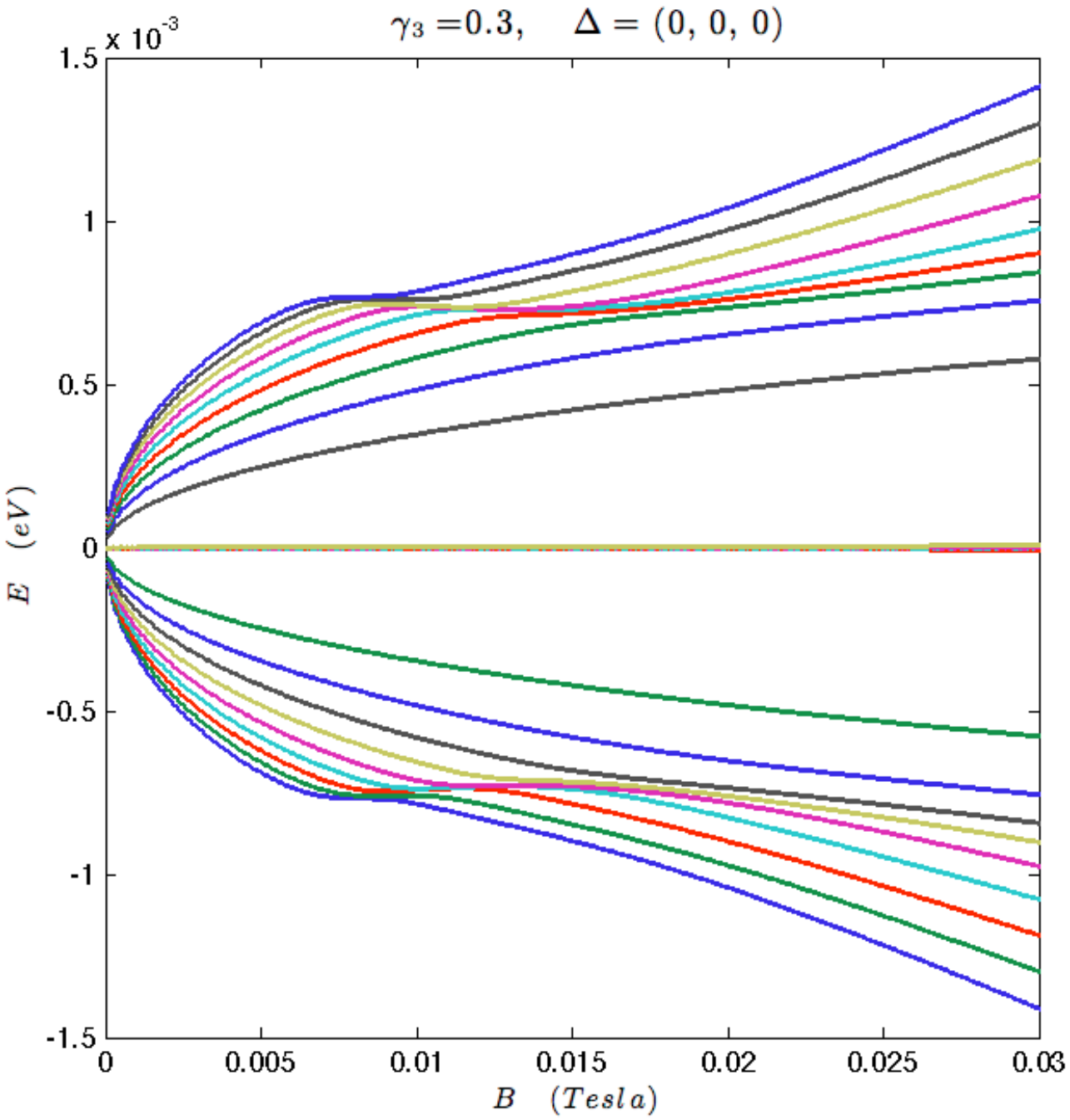


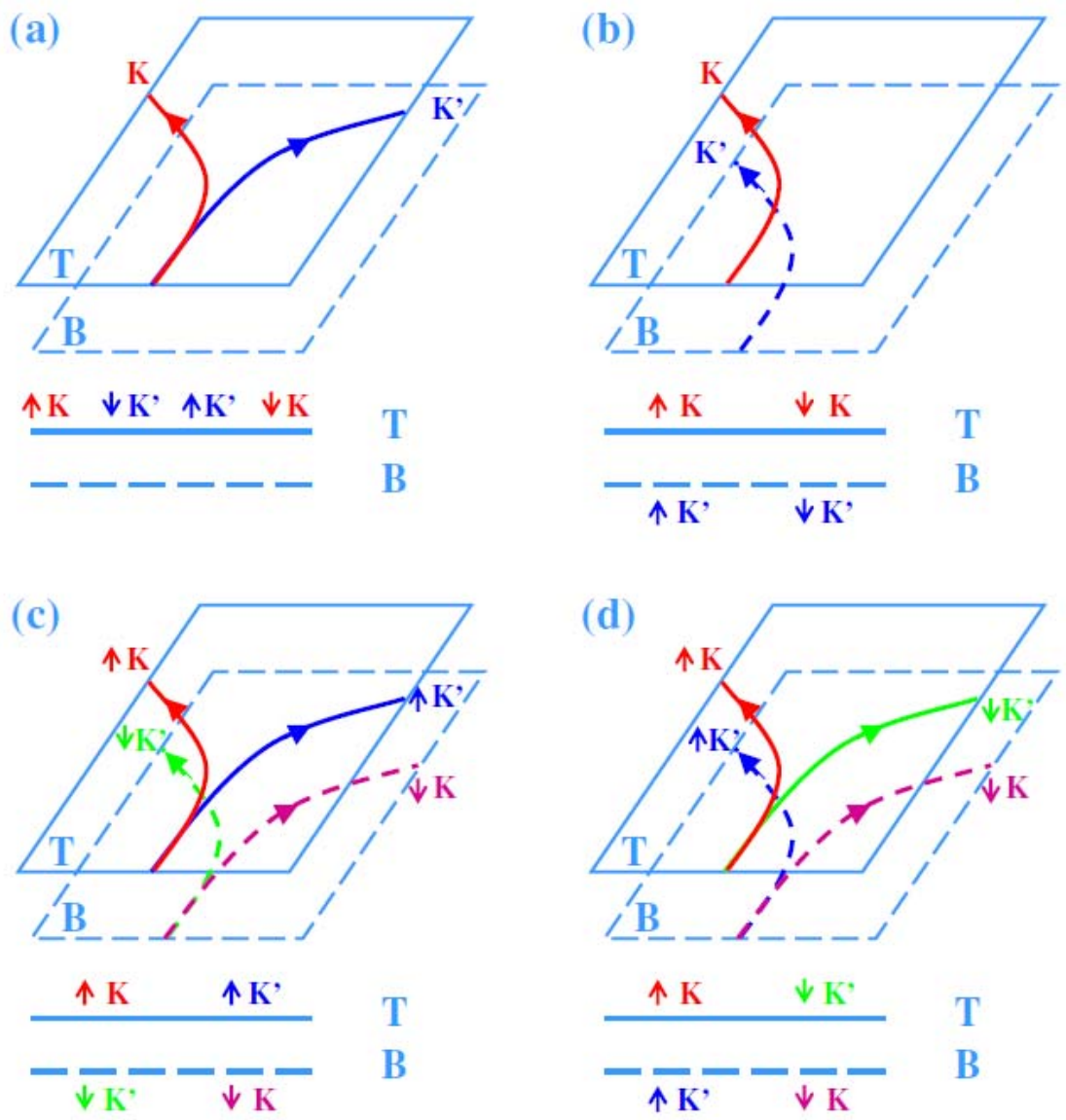


tilted






$\gamma_3 = 0.3, \quad \Delta = (0, 0, 0)$
x 10^-3
1.5
1
0.5
0
-0.5
-1
-1.5
E (eV)
0
0.005
0.01
0.015
0.02
0.025
0.03
B (Tesla)



(a) = `ps ferro'
σ_{Hall} =0

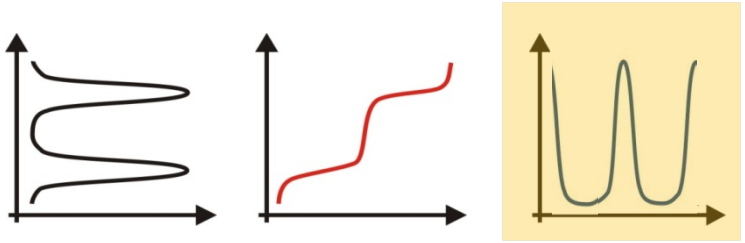
(b) = `anomalous Hall insulator' (Levitov)
$\sigma_{Hall} \neq 0$

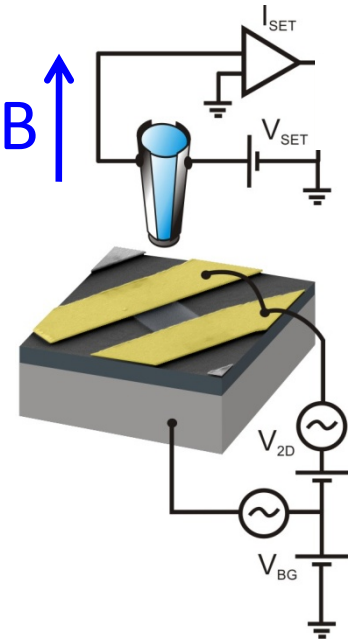
(c)= `layer AF insulator'
σ_{Hall} =0

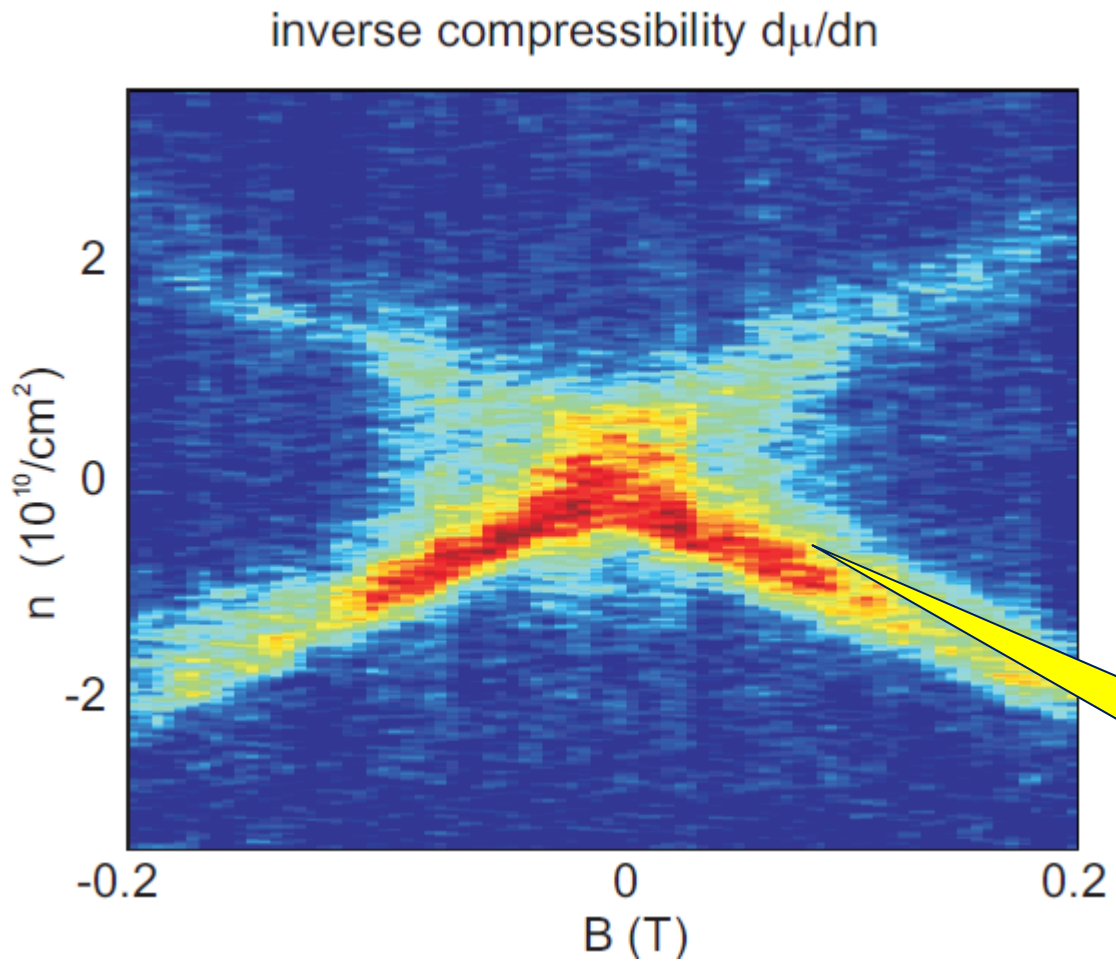
(d) = `spin Hall insulator'
σ_{Hall} =0

R. Nandkishore & Levitov PRB (2010)
Fan Zhang *et al.* PRL (2011)

Local inverse compressibility, B → 0 T





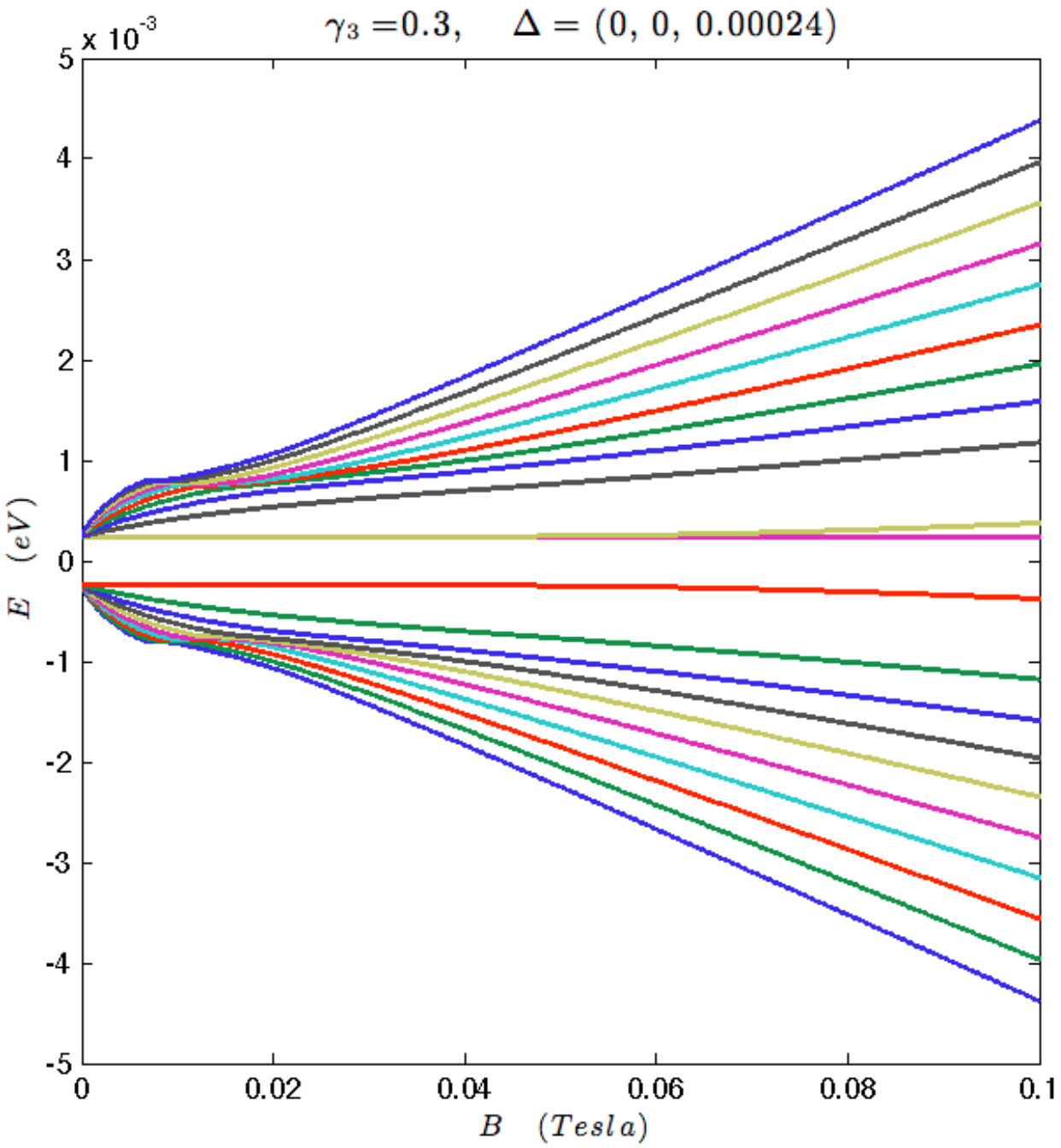


Feldman *et al.* Nature Physics (2009)

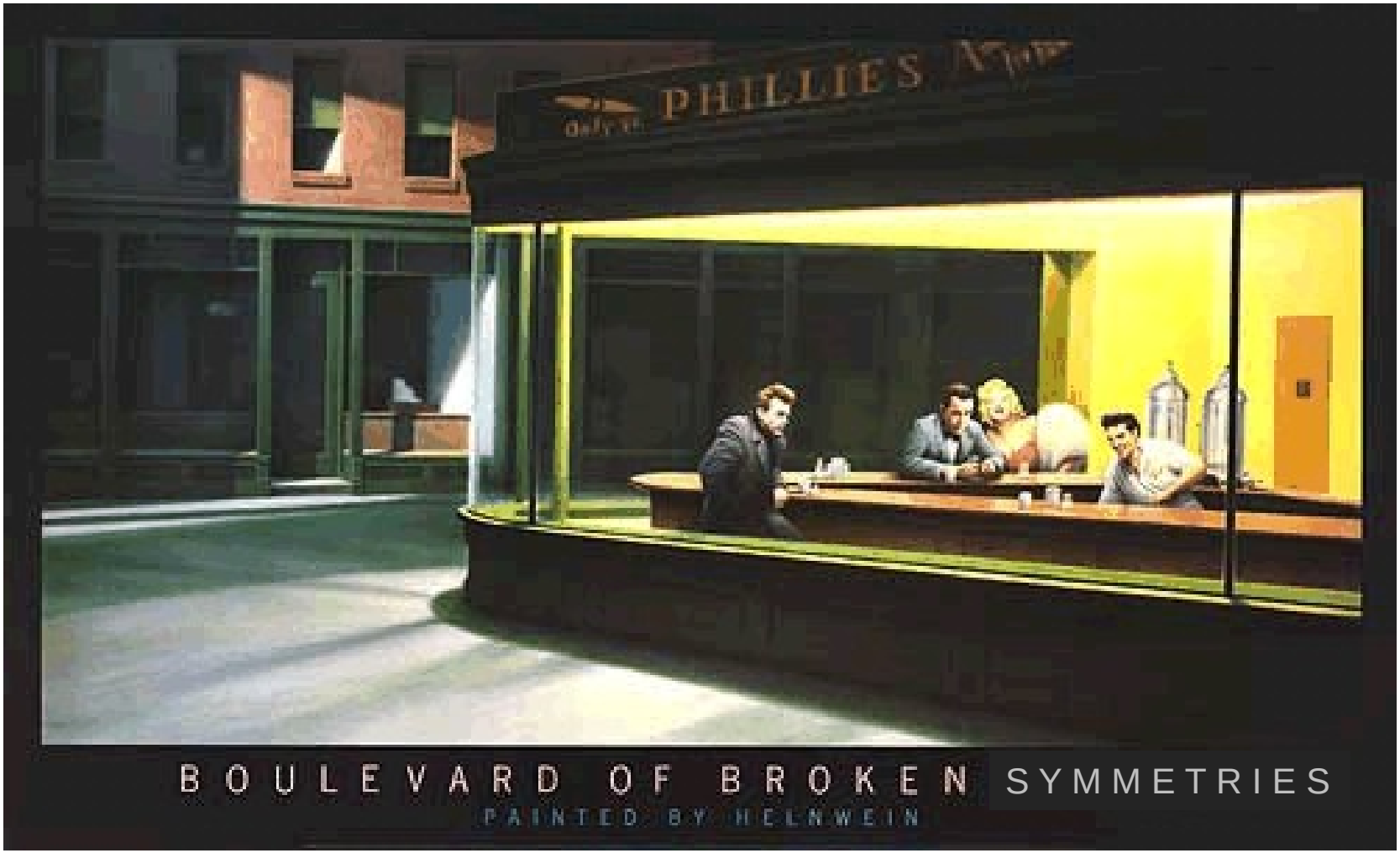
Martin *et al.* PRL (2010)

σ_{Hall} ~ dn/dB (Streda)

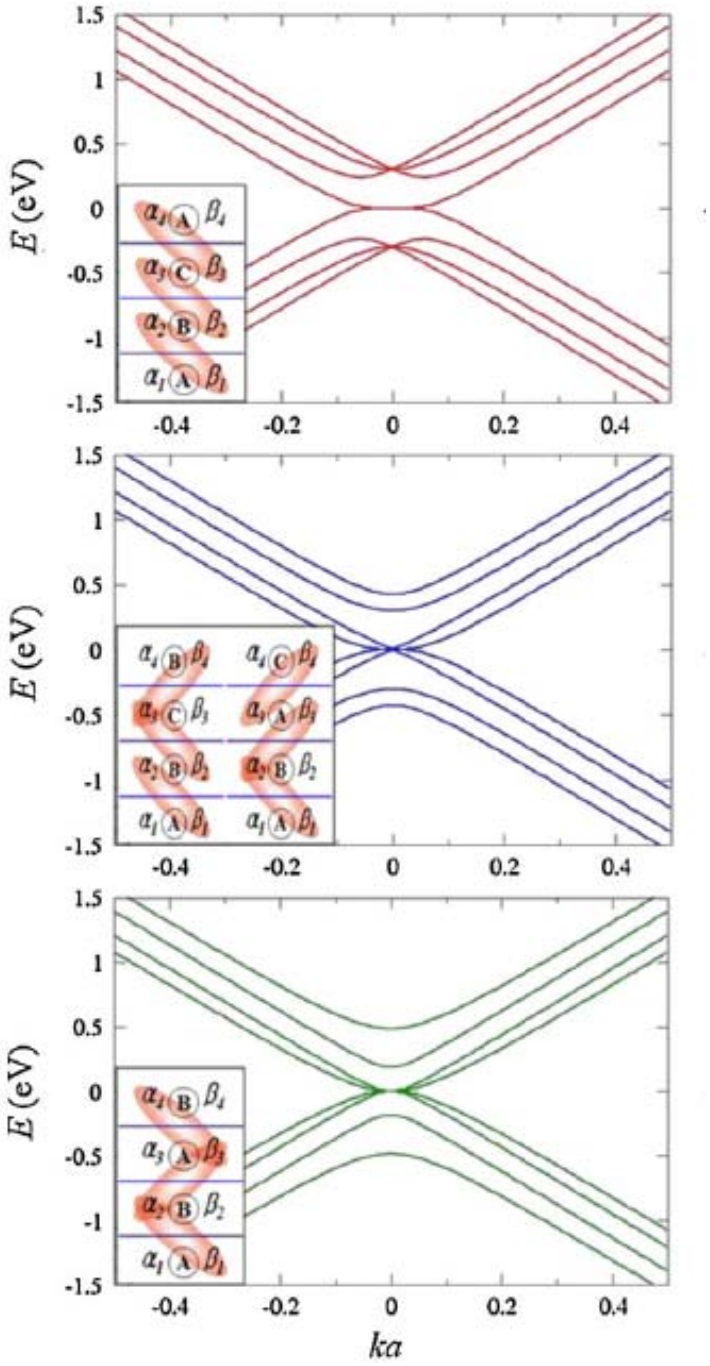
Order Parameter Dependence of Landau Levels



Condensed Matter Physics



(apologies to Adriaan Schakel)



N-layer ABC Graphene

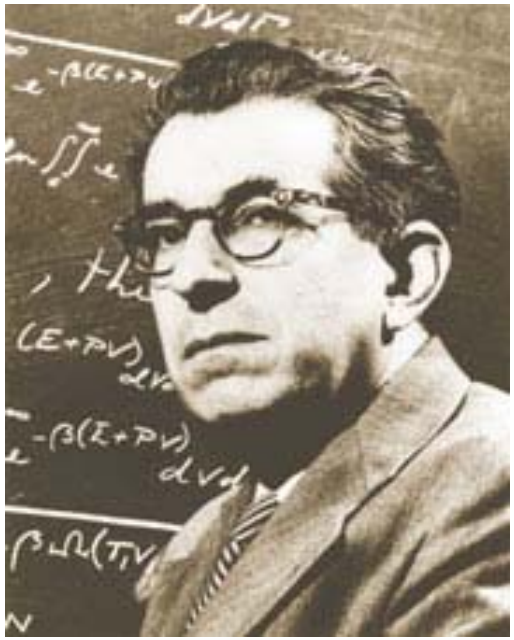
Min and AHM,
PRB (2008), PRL (2009)
Fan et al. PRB (2010)

- Spontaneous Hall States Bilayer Graphene
- Doped Bilayer Graphene

On the Problem of the Molecular Theory of Superconductivity*

F. LONDON
Duke University, Durham, North Carolina
(Received April 25, 1948)

Werner Heisenberg



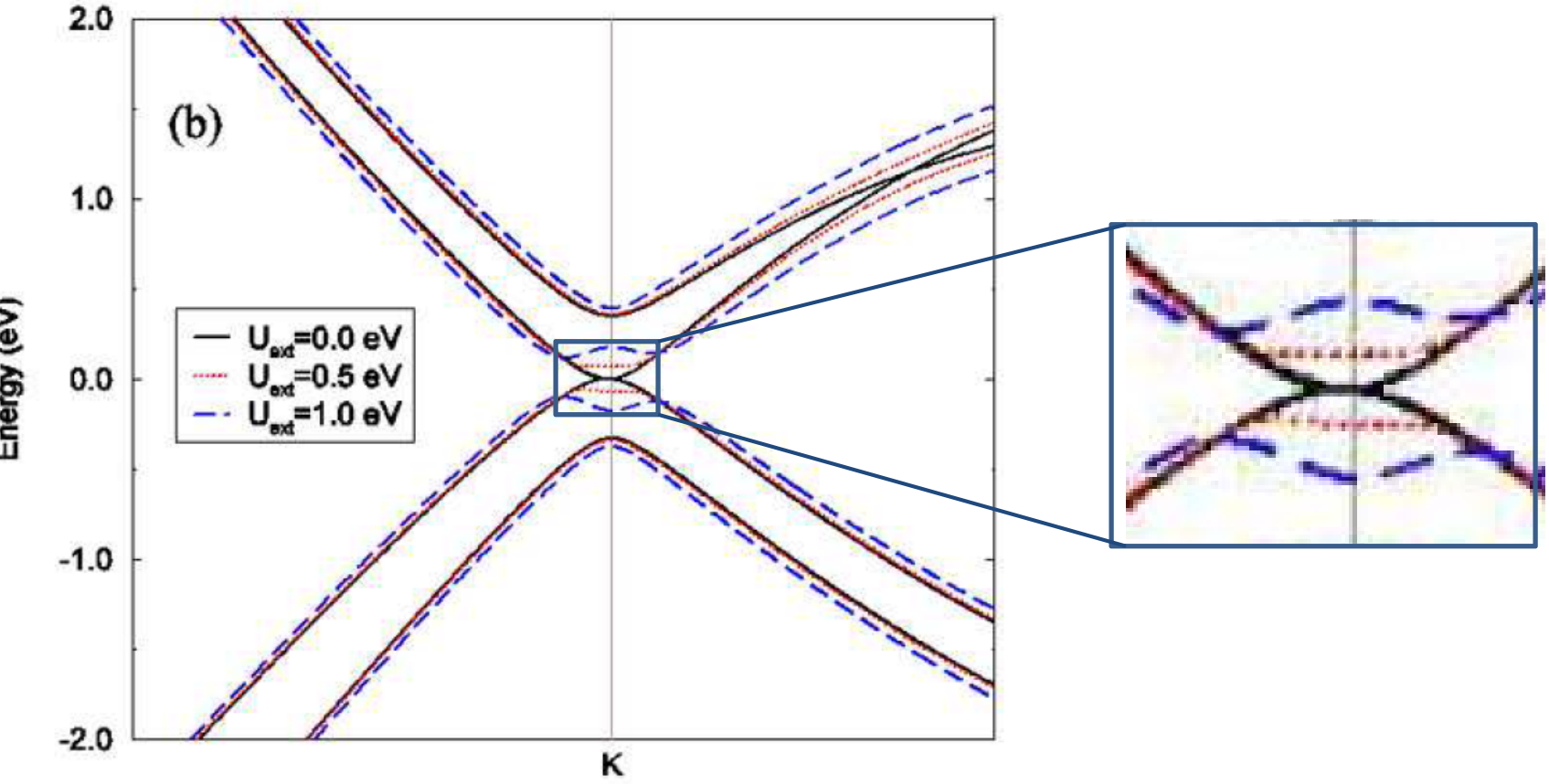
Fritz London

$$I_{kl} = 4\pi h^2 e^2 / V \left| p_k - p_l \right|^2$$

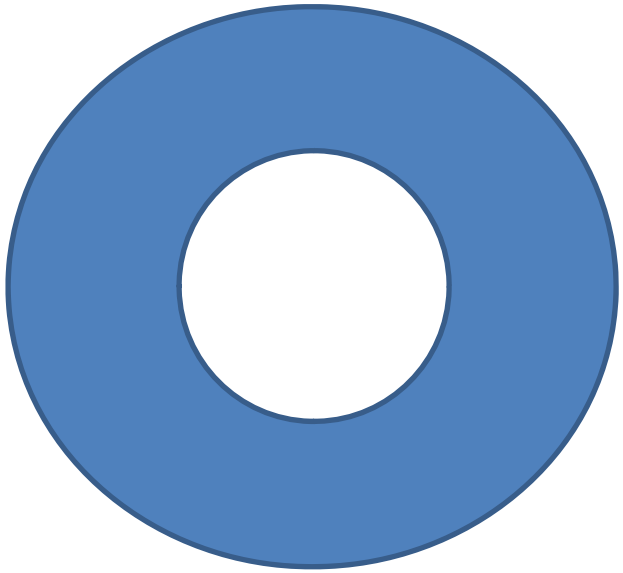
We assembled indications which suggest that it is most probably the exchange interaction associated with the Coulomb field of the electrons which is responsible for this "condensation in momentum space." Ferromagnetism and superconductivity would then be considered as two opposite limiting cases of the same effect, depending on whether the exchange interaction competing with the zero-point energy promotes parallel orientation of the electronic spins or a coordination of the translational momentum in a state of vanishing total spin.

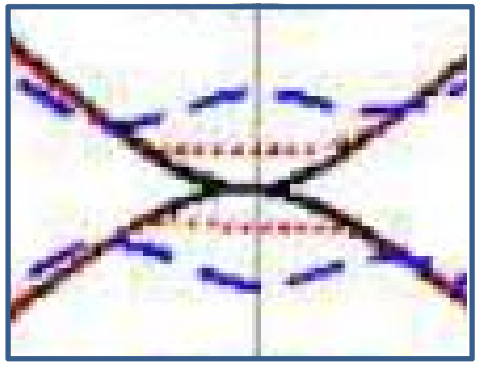
F. London Phys. Rev. **74** (1948)

Doped Bilayer Graphene

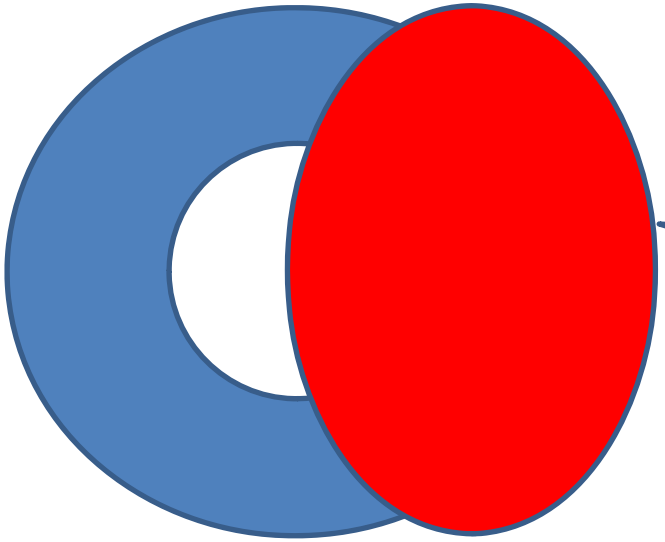


Doped Bilayer Graphene





Doped Bilayer Graphene



F. London's Spontaneous Current State

We assembled indications which suggest that it is most probably the exchange interaction associated with the Coulomb field of the electrons which is responsible for this "condensation in momentum space." Ferromagnetism and superconductivity would then be considered as two opposite limiting cases of the same effect, depending on whether the exchange interaction competing with the zero-point energy promotes parallel orientation of the electronic spins or a coordination of the translational momentum in a state of vanishing total spin.

F. London Phys. Rev. **74** (1948)

Thank you!

SWAN

Allan MacDonald – UT Austin